Deep Occlusion Reasoning for Multi-Camera Multi-Target Detection

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Abstract

People detection in single 2D images has improved greatly in recent years. However, comparatively little of this progress has percolated into multi-camera multi-people tracking algorithms, whose performance still degrades severely when scenes become very crowded. In this work, we introduce a new architecture that combines Convolutional Neural Nets and Conditional Random Fields to explicitly model those ambiguities. One of its key ingredients are high-order CRF terms that model potential occlusions and give our approach its robustness even when many people are present. Our model is trained end-to-end and we show that it outperforms several state-of-the-art algorithms on challenging scenes.

1. Introduction

Multi-Camera Multi-Target Tracking (MCMT) algorithms have long been effective at tracking people in complex environments. Before the emergence of Deep Learning, some of the most effective methods relied on simple background subtraction, geometric and sparsity constraints, and occlusion reasoning [12, 6, 1]. Given the limited discriminative power of background subtraction, they work surprisingly well as long as there are not too many people in the scene. However, their performance degrades as people density increases, making the background subtraction used as input less and less informative.

Since then, Deep Learning based people detection algorithms in single images [23, 19, 28] have become among the most effective [28]. However, their power has only rarely been leveraged for MCMT purposes. Some recent algorithms, such as the one of [27], attempt to do so by first detecting people in single images, projecting the detections into a common reference-frame, and finally putting them into correspondence to achieve 3D localization and eliminate false positives. As shown in Fig. 1, this is prone to errors for two reasons. First, projection in the reference frame is inaccurate, especially when the 2D detector has not been specifically trained for that purpose. Second, the projection is usually preceded by Non Maximum Suppression (NMS) on the output of the 2D detector, which does not take into account the multi-camera geometry to resolve ambiguities.

Ideally, the power of Deep Learning should be combined with occlusion reasoning much earlier in the detection process than is normally done. To this end, we designed a joint CNN/CRF model whose posterior distribution can be approximated by Mean-Field inference using standard differentiable operations. Our model is trainable end-to-end and can be used in both supervised and unsupervised scenarios.

More specifically, we reason on a discretized ground plane in which detections are represented by boolean variables. The CRF is defined as a sum of innovative high-order terms whose values are computed by measuring the discrepancy between the predictions of a generative model that accounts for occlusions and those of a CNN that can infer that certain image patches look like specific body parts. To these terms, we add unary and pairwise ones to increase robustness and model physical repulsion constraints.

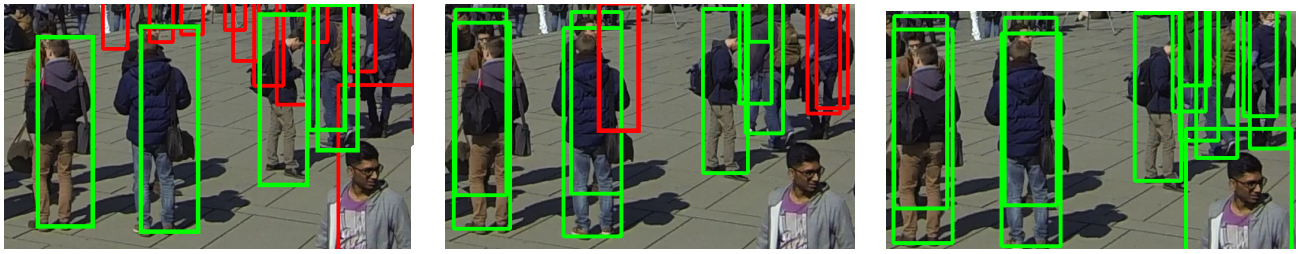
To summarize, our contribution is a joint CNN/CRF pipeline that performs detection for MCMT purposes in such a way that NMS is not required. Because it explicitly models occlusions, our algorithm operates robustly even in crowded scenes. Furthermore, it outputs probabilities of presence on the ground plane, as opposed to binary detections, which can then be linked into full trajectories using a simple flow-based approach [6].

2. Related Work

In this section, we first discuss briefly recent Deep Learning approaches to people detection in single images. We then move on to multi-image algorithms and techniques for combining CNNs and CRFs.

2.1. Deep Monocular Detection

As in many other domains, CNN-based algorithms [23, 19, 22] have become very good for people detection in sin-



RCNN-2D/3D **POM-CNN** **Ours**

Figure 1. Multi-camera detection in a crowded scene. Even though there are 7 cameras with overlapping fields of view, baselines inspired by earlier approaches—-**RCNN-2D/3D** by [27] and **POM-CNN** by [12], as described in Section 7.2—both generate false positives denoted by red rectangles and miss or misplace a number of people, whereas ours does not. This example is representative of the algorithm's behavior and is best viewed in color. Please see supplementary material for results on a video sequence.

gle images and achieve state-of-the-art performance [28]. Algorithms in this class usually first propose potential candidate bounding boxes with scores assigned to them. They then perform Non-Maximum Suppression (NMS) and return a final set of candidates. The very popular method of [23] performs both steps in a single CNN pass through the image. It returns a feature map in which a feature vector of constant dimension is associated to each image pixel. For any 2D bounding-box of any size in that image, a feature vector of any arbitrary dimension can then be computed using Region Of Interest (ROI) pooling and fed to a classifier to assess whether the bounding box does indeed correspond to a true detection.

While this algorithm has demonstrated its worth on many benchmarks, it can fail in crowded scenes such as the one of Fig. 1. This is perennial problem of single-image detectors when people occlude each other severely. One solution to this problem is to rely on cameras with overlapping fields of view, as discussed below.

2.2. Multi-Camera Pedestrian Detection

Here, we distinguish between recent algorithms that rely on Deep Learning but do not explicitly account for occlusions and older ones that model occlusions and geometry but appeared before the Deep Learning became popular. Our approach can be understood as a way to bring together their respective strengths.

The recent algorithm of [27] runs a monocular detector similar to the one of [23] on multiple views and infers people ground locations from the resulting detections. However, this method is prone to errors both because the 2D detections are performed independently of each other and because combining them by projecting them onto the ground plane involves reprojection errors and ignores occlusions. Yet, it is representative of the current MCMT state-of-the-art and is benchmarked against much older algorithms [12, 6] that rely on background subtraction instead of a Deep Learning approach.

These older algorithms use multiple cameras with overlapping fields of view to leverage geometrical or appearance consistency across views to resolve the ambiguities that arise in crowded scenes and obtain accurate 3D localisation [12, 1, 21]. They rely on Bayesian inference and graphical models to enforce detection sparsity. For example, the Probabilisitic Occupancy Map (POM) approach [12] takes background subtraction images as input and relies on Mean Field inference to compute probabilities of presence in the ground plane. More specifically, given several cameras with overlapping fields of view of a discretized ground plane, POM first performs background subtraction. It then uses a generative model that represents humans as simple rectangles in order to create synthetic ideal images that would be observed if people were at given locations. Under this model of the image given the true occupancy, it approximates the probabilities of occupancy at every location using Mean Field inference. Because the generative model explicitly accounts for occlusions, POM is robust and often performs well. But it relies on background subtraction results as its only input, which is not discriminative enough when the people density increases, as shown in Fig. 1. The algorithm of [1] operates on similar principles as POM but introduces more sophisticated human templates. Since it also relies on background subtraction, it is subject to the same limitations when the people density increases. And so is the algorithm of [21] that introduces a more complex Bayesian model to enhance the results of [1].

2.3. Combining CNNs and CRFs

Using a CNN to compute potentials for a Conditional Random Field (CRF) and training them jointly for structured prediction purposes has received much attention in recent years [18, 10, 11, 29, 2, 15, 17, 3]. However, properly training the CRFs remains difficult because many interesting models yield intractable inference problems. A popular workaround is to optimize the CRF potentials so as to minimize a loss defined on the output of an inference algorithm. Back Mean-Field [11, 29, 2, 17] has emerged as a promising way to do this. It relies on the fact that the update steps

during Mean-Field inference are continuous and parallelizable [4]. It is therefore possible to represent these operations as additional layers in a Neural Network and back-propagate through it. So far, this method has mostly been demonstrated either for toy problems or for semantic segmentation with attractive potentials, whereas our approach also requires repulsive potentials.

3. Modeling Occlusions in a CNN Framework

The core motivation behind our approach is to properly handle occlusions, while still leveraging the power of CNNs and on perfectly calibrated, fixed cameras. To do so, we must model the interactions between multiple people who occlude each other but may not be physically close to each other. Our solution is to introduce an *observation space*; a generative model for observations given where people are located in the ground plane; and a discriminative model that predicts expected observations from the images. We then define a loss function that measures how different the CNN predictions are from those generated by the model. Finally, we use a Mean-Field approach with respect to probabilities of presence in the ground plane to minimize this loss. We cast this computation in terms of minimizing the energy of a Conditional Random Field in which the interactions between nodes are non-local because the people who occlude each other may not be physically close, which requires long range high-order terms.

In the remainder of this section, we first introduce the required notations to formalize our model. We then define a CRF that only involves high-order interaction potentials. Finally, we describe a more complete one that also relies on unary and pairwise terms.

3.1. Notations

We discretize the ground plane in grid cells and introduce Boolean variables that denote the presence or absence of someone in the cell. Let us therefore consider a discretized ground plane containing N locations. Let Z_i be the boolean variable that denotes the presence of someone at location i. Let us assume we are given C RGB images I^c of size $H^c \times W^c$ from multiple views $1 \leq c \leq C$ and $I = \{I^1, \ldots, I^C\}$. For each ground plane location i and camera c, let the smallest rectangular zone containing the 2D projection of a human-sized 3D cylinder located at i be defined by its top-left and bottom-right coordinates T_i^c and B_i^c. For a pixel $k \in \{1, \ldots, H^c\} \times \{1, \ldots, W^c\}$, let L_k^c be the set of such projections that contain k.

We also introduce a CNN that defines an operator $\mathcal{F}(\cdot; \theta_F)$, which takes as input the RGB image of camera c and outputs a feature map $\mathcal{F}^c = \mathcal{F}(I^c; \theta_F)$, where θ_F denotes the network's parameters. It contains a d-dimensional vector $\mathcal{F}_k^c$ for each pixel k.

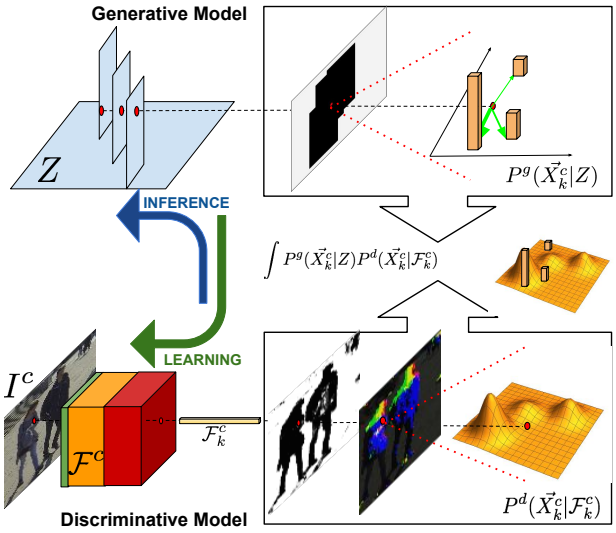


Figure 2. Schematic representation of our High-Order potentials as described in Section 3.2.2. See supplementary material.

3.2. High-Order CRF

We take the energy of our CRF to be a sum of High-Order potentials $\psi_{\text{h}}^{c,k}$, one for each pixel. They handle jointly detection, and occlusion reasoning while removing the need for Non-Maximum Suppression. Each of these potentials use Probability Product Kernels [13] to represent the agreement between a generative model and a discriminative model over the *observation space*, at a given pixel, as depicted in Fig. 2. We therefore write

$$P(Z; I) = \frac{1}{\mathcal{Z}} \exp \psi_{\text{h}}(Z; \mathcal{F}(I; \theta_F)) , \quad (1)$$

$$\psi_{\text{h}}(Z; \mathcal{F}) = \sum_{1 \leq c \leq C, k \in \{1,\ldots,H^c\} \times \{1,\ldots,W^c\}} \psi_{\text{h}}^{c,k}(Z; \mathcal{F}_k^c) .$$

Assuming we know the values of the occupancy variables Z, the generative model computes distributions over the set of *observations*. For each pixel in each image, it computes a distribution over possible 2D vectors representing observed bounding-box regions. To this end, it considers the locations such that $Z_i = 1$, crossing the corresponding line of sight, and uses the simple generative occlusion model described below. This results in images whose pixels are vectors representing a distribution of 2D vectors, the observations, as depicted in the top row of Fig 2. Our discriminative model relies on a CNN which tries to predict similar distributions of 2D vectors, directly by looking at the image. For ease of understanding, we first present in more details a simple version of our High-Order potentials $\psi_{\text{h}}^{c,k}$. It assumes that our observations are zeros and ones at every pixel. The discriminative model therefore acts much as the background subtraction algorithms used in [12] did. We then extend them to take into account the 2D vector output of our discriminative model.

3.2.1 Simple Generative Model

We first introduce a binary *observation* variable $X_k^c \in \{0,1\}$ over which we define two distributions P^g and P^d produced by the generative and discriminative model respectively. We take the distribution P^g to be

$$\begin{aligned} P^g(X_k^c = 1|Z) = 0, \text{ if } Z_i = 0 \; \forall i \in L_k^c \; , \\ P^g(X_k^c = 1|Z) = 1 \text{ otherwise,} \end{aligned} \tag{2}$$

and the discriminative one P^d to be $P^d(X_k^c|\mathcal{F}_k^c) = f_{\text{b}}(\mathcal{F}_k^c;\theta_b)$, where F_k^c is the d-dimensional feature vector associated to pixel k introduced above and f_{b} is a Multi-Layer Perceptron (MLP) with weights θ_b. In other words, f_{b} plays the role of a CNN-based semantic segmentor or background-subtraction.

For each pixel, we then take the high-order potential to be the dot product between the distributions

$$\psi_{\text{h}}^{c,k}(Z;\mathcal{F}_k^c) = \mu_{\text{h}} \log \int_{X_k^c \in \{0,1\}} P^g(X_k^c|\{Z_i\}_{i\in L_k^c}) P^d(X_k^c|\mathcal{F}_k^c) \; , \tag{3}$$

as in the probability product kernel method of [13]. Intuitively, $\psi_{\text{h}}^{c,k}$ is high when the segmentation produced by the network matches the projection of the detections in each camera plane using the simple generative model of Eq. 2. μ_{h} is an energy scaling parameter.

3.2.2 Full Generative Model

The above model correctly accounts for occlusions and geometry but ignores much image information by focusing on background / foreground decisions. To refine it, we model the part of the bounding-box a pixel belongs to rather than just the fact that it belongs to a bounding-box. To this end, we redefine the Boolean auxiliary variable X_k^c as

$$\vec{X}_k^c \in \{0\} \cup \mathbb{R}^2 \; , \tag{4}$$

where the label 0 represents background as before, and a label in $\mathbb{R}^2$ denotes the displacement with respect to the center of the body of the visible person at this pixel location.

To extend the simple model and account for what part of a bounding-box pixel k belongs to if it does, we sample from the distribution $P^g(\vec{X}_k^c|\{Z_i\}_{i\in L_k^c})$. To this end, let us assume without loss of generality that the L_k^c are ordered by increasing distance to the camera, as shown in the top left corner of Fig. 2. We initialize the variables $\vec{X}_k^c$ to 0. Then, for each i in L_k^c such that $Z_i = 1$, we draw a boolean random variable O_i with fixed expectancy o. If $O_i = 1$, then

$$\begin{aligned} \vec{X}_k &= \vec{x}_k^i \; , \\ &= \left(\frac{k_x - 0.5(T_{i\,x}^c + B_{i\,x}^c)}{B_{i\,x}^c - T_{i\,x}^c}; \frac{k_y - 0.5(T_{i\,y}^c + B_{i\,y}^c)}{B_{i\,y}^c - T_{i\,y}^c} \right) \; , \end{aligned} \tag{5}$$

that is, the relative location of pixel k with respect to the projection of detection i in camera c, as depicted in the upper right corner of Fig. 2.

We define the distribution $P^d(\vec{X}_k|\mathcal{F}_k^c)$ as an M-Modal Gaussian Mixture

$$\begin{aligned} P^d(\vec{X}_k = 0) &= f_{\text{b}}(\mathcal{F}_k^c;\theta_b) \; , \\ P^d(\vec{X}_k|\vec{X}_k \neq 0) &= \sum_{1\leq m\leq M} f_{\text{h}}(\mathcal{F}_k^c;\theta_{\text{h}})_m \mathcal{N}(\vec{X}_k - \alpha_m;\sigma_m) \; , \end{aligned} \tag{6}$$

as depicted in the bottom right corner of Fig. 2. As a result, $P^d(\vec{X}_k = 0)$ is the same as in the simple model but $P^d(\vec{X}_k|\vec{X}_k \neq 0)$ encodes more information. (α_m, σ_m) are Gaussian parameters learned for each mode m. f_{h} is a MLP parametrized by θ_{h} that outputs M normalized real probabilities where M is a meta-parameter of our model. Similarly, $f_{\text{b}}(\mathcal{F}_k^c;\theta_b)$ is a background probability.

Finally, as in Eq. 3, we take our complete potential to be

$$\psi_{\text{h}}^{c,k}(Z;\mathcal{F}_k^c) = \mu_{\text{h}} \log \int_{\vec{X}_k \in \{0\}\cup\mathbb{R}^2} P^g(\vec{X}_k|\{Z_i\}_{i\in L_k^c}) P^d(\vec{X}_k|\mathcal{F}_k^c) \; . \tag{7}$$

3.3. Complete CRF

To increase the robustness of our CRF, we have found it effective to add, to the high-order potentials of Eq. 1, unary and pairwise ones to exploit additional image information. We therefore write our complete CRF model as

$$\begin{aligned} P(Z;I) &= \frac{1}{\mathcal{Z}} \exp \psi(Z;\mathcal{F}) \; , \\ \psi(Z;\mathcal{F}) &= \psi_{\text{h}}(Z;\mathcal{F}) + \sum_{i\leq N} \psi_{\text{u}}^i(Z_i;\mathcal{F}) + \sum_{i\leq N, j\leq N} \psi_{\text{p}}(Z_i,Z_j), \end{aligned} \tag{8}$$

where ψ_{h} is the high-order CRF of Eq. 1, the ψ_{u}^i are unary potentials, and ψ_{p} pairwise ones, which we describe below.

3.3.1 Unaries

The purpose of our unary potentials is to provide a prior probability of presence at a given location on the ground, before considering the occlusion effect and non maximum suppression. For each location i and camera c, we use a CNN $f_{\text{u}}(T_i^c, B_i^c, \mathcal{F}^c)$, with parameters θ_{u}, which outputs a probability of presence of a person at location i. f_{u} works by extracting a fixed size feature vector from the rectangular region defined by T_i^c, B_i^c in $\mathcal{F}^c$, using an ROI pooling layer [23]. A detection probability is finally estimated using an MLP. Estimates from the multiple cameras are pooled through a max operation

$$\psi_{\text{u}}^i(Z_i;\mathcal{F}) = \mu_{\text{u}} Z_i \max_c \log \frac{f_{\text{u}}(T_i^c, B_i^c, \mathcal{F}_c)}{1 - f_{\text{u}}(T_i^c, B_i^c, \mathcal{F}_c)} \; , \tag{9}$$

where μ_{u} is a scalar that controls the importance of unary terms compared to others.

3.3.2 Pairwise

The purpose of our pairwise potentials is to represent the fact that two people are unlikely to stand too close to each others. For all pairs of locations (i, j), let $E_{\text{p}}^{i,j} = E_{\text{p}}[|x_i - x_j|; |y_i - y_j|]$, where E_{p} is a 2D kernel function of of predefined size. We write

$$\psi_{\text{p}}(Z_i, Z_j) = -E_{\text{p}}^{i,j} Z_i Z_j \tag{10}$$

for locations that are closer to each other than a predefined distance and 0 otherwise.

4. Inference and Derivation

Given the CRF of Eq. 8 and assuming all parameters known, finding out where people are in the ground plane amounts to minimizing ψ with respect to Z, the vector of binary variables that indicates which ground locations contain someone, which amounts to computing a Maximum-a-Posteriori of the posterior P. Instead of doing so directly, which would be intractable, we use Mean-Field inference [26] to approximate P by a fully-factorised distribution Q. As in [12], this produces a Probability Occupancy Map, that is, a probability of presence $Q(Z_i = 1)$, at each location, such as the one depicted by Fig. 3.

(a) (b)

Figure 3. Output. (a) Given a set of images of the same scene, ours algorithm produces a Probabilistic Occupancy Map, that is, a probability of presence at each location of the ground plane. Red values indicate probabilities close to 1 and blue ones values close to zero. (b) Because the probabilities are very peaked, they can easily be thresholded to produce detections whose projections are the green boxes in the original image(s).

To perform this minimization, we rely on the natural-gradient descent scheme of [4]. It involves taking gradient steps that are proportional to

$$\nabla_{\eta_i} = \mathbb{E}_Q\left[(\psi(Z, \mathcal{F}))\,|Z_i = 1\right] - \mathbb{E}_Q\left[(\psi(Z, \mathcal{F}))\,|Z_i = 0\right], \tag{11}$$

for each location i. The contribution to ∇_{η_i} of the unaries derives straightforwardly from Eq. 9. Similarly, the one of the pairwise potentials of Eq. 10 is

$$\begin{aligned}(\nabla_{\eta_i})_{\text{p}} &= -\sum_j E_{\text{p}}^{i,j} Q_j(Z_j = 1)\,, \\ &= -\sum_j E_{\text{p}}[|x_i - x_j|, |y_i - y_j|] Q_j(Z_j = 1)\,,\end{aligned} \tag{12}$$

which can be implemented as a convolution over the current estimate of the probabilistic occupancy map Q with the two dimensional kernel $E_{\text{p}}[.,.]$. This makes it easy to unroll the inference steps using a Deep-Learning framework.

Formulating the contributions of the higher-order terms of Eq. 7 is more involved and requires simplifications. We first approximate the Gaussians used in Eq. 6 by a function whose value is 1 in B_m and ϵ elsewhere, where B_m is the rectangle of center α_m and half-size $3\sigma_m$. Note that this approximation is only used for inference purposes, and that during training, it keeps its original Gaussian form. We then threshold the Gaussian weights f_{h} resulting in the binary approximation $\widetilde{f}_{\text{h}}$. This yields a binary approximation $\widetilde{P}^d(\vec{X}_k)$ of $P^d(\vec{X}_k)$. Note that the corresponding approximate potential $\widetilde{\psi}_{\text{h}}^{c,k}(Z, \mathcal{F}_k^c)$ can be either $O(\log \epsilon)$, if $P(\vec{X}_k, b_k = 1; Z) = 0$ for all $\vec{X}_k$ such that $P^d(\vec{X}_k) > \epsilon$ or $O(\log(1))$. Hence, the configurations where $\psi_{\text{h}}^{c,k}(Z, \mathcal{F}_k^c) = O(\log \epsilon)$ will dominate the others when computing the expectancies. This yields the approximation of Eq. 11,

$$\widetilde{\nabla \eta_i} = -C(\mathbb{E}_Q\left[\Delta(Z)|Z_i = 1\right] - \mathbb{E}_Q\left[\Delta(Z)|Z_i = 0\right])\,, \tag{13}$$

where $C = -log\epsilon$ is a constant and $\Delta(Z)$ is a binary random variable, which takes value 1 if $\widetilde{\psi}_{\text{h}}^{c,k}(Z, \mathcal{F}_k^c) = 0$, and 0 otherwise. Note that $\psi_{\text{h}}^{c,k}(Z, \mathcal{F}_k^c) = O(\log(1))$ iff

$$\exists i \leq N, m \leq M \text{ s.t } \widetilde{f}_{\text{h}}(\mathcal{F}_k^c; \theta_{\text{h}})_m = 1 \text{ and } \vec{X}_k^i \in B_m\,. \tag{14}$$

This means that for each pixel k, given a thresholded output from the network $\widetilde{f}_{\text{h}}(\mathcal{F}_k^c; \theta_{\text{h}})$, we obtain a list of *compatible explanations* $\mathcal{C}_k \subset \{1, \ldots, N\}$ such that pixel k defines a very simple pattern-based potential of the form 1 if $Z_i = 0\ \forall i \in \mathcal{C}_k$, 0 otherwise, which is similar to the potentials used in the Mean-Fields algorithms of [25, 12, 16, 2, 5]. In the supplementary material, we see how this operation can be implemented efficiently using common Deep-Learning operations and integral-images.

5. Training

We now show how our model can be trained first in a supervised manner and then in an unsupervised one.

5.1. Supervised Training

Let us first assume that we observe D data points $(Z^0, I^0), \ldots, (Z^D, I^D)$, where I^d represents a multi-view image and Z^d the corresponding ground truth presences. The purpose of training is then to optimize the network parameters $\theta_F, \theta_{\text{u}}, \theta_{\text{h}}$ defined in Sections 3.1, 3.3.1 and 3.2.2 respectively, the gaussian parameters α, σ of Eq. 6 and the energy-scaling meta-parameters $\mu_{\text{u}}, \mu_{\text{h}}$ of Eqs. 9 and 3 to maximize $\sum_{d \leq D} \log P(Z^d; I^d)$. It cannot be done directly using Eq. 8 because computing the partition function $\mathcal{Z}$ is intractable.

Back Mean-Field An increasingly popular work-around is to optimize the above-mentioned parameters to ensure that the output of the Mean-Field inference fits the ground truth. In other terms, let $Q_{\theta_F,\theta_\text{u},\theta_\text{h},\alpha,\sigma}(Z;I)$ be the distribution obtained after inference . We look for

$$\underset{\theta_F,\theta_\text{u},\theta_\text{h},\alpha,\sigma}{\text{argmax}} \sum_{(Z^d,I^d)} \log Q_{\theta_F,\theta_\text{u},\theta_\text{h},\alpha,\sigma}(Z=Z^d;I^d) \ . \quad (15)$$

Since $Q_{\theta_F,\theta_\text{u},\theta_\text{h},\alpha,\sigma}(Z = Z^d;I^d)$ is computed via a sequence of operations which are all differentiable with respect to the parameters θ_F, θ_u, and θ_h, it is therefore possible to solve Eq. 15 by stochastic gradient descent [11, 29].

Pre-training However, it still remains difficult to optimize the whole model from scratch. We therefore pre-train our potentials separately before end-to-end fine-tuning. More precisely, the CNN f_u that appears in the unary terms of Eq. 9 is trained as a standard classifier that gives the probability of presence at a given location, given the projection of the corresponding bounding-box in each camera view. For each data point, this leaves the high-order terms for which we need to optimize

$$\sum_c \sum_{k\in\mathcal{P}_c} \log(\psi_\text{h}^{c,k}(Z^d,\mathcal{F}_k^c)) \ , \quad (16)$$

with respect to the parameters of the Gaussian Mixture network θ_h, α, and σ. We use Jensen's inequality to take our generative distribution P^g out of the integral in Eq. 7 and approximate it by random sampling procedure described in Section 3.2.2. We rewrite the set of samples for $\vec{X}_k^c$ from all the pixels from all the cameras from all the data-points as $S(Z^0,\ldots,Z^D)$. The optimization objective of Eq. 16 can then be rewritten as

$$\sum_{\vec{x}_s\in S(Z^0,\ldots,Z^D)} \log(P^d(\vec{x}_s|\mathcal{F}_k^c,\theta_\text{h},\alpha,\sigma)) \ , \quad (17)$$

which is optimized by alternating a standard stochastic gradient descent for the θ_h parameters and a closed form batch optimization for α,σ. This procedure is similar to one often used to fit a Mixture of Gaussians, except that, during the E-Step, instead of computing the class probabilities directly to increase the likelihood, we optimise the parameters of the network through gradient descent. More details are provided in the supplementary material.

This pre-training strategy creates potentials which are reasonable but not designed to be commensurate with each others. We therefore need to choose the two energy parameters scalars μ_u, and μ_h, via grid-search in order to optimize the relative weights of Unary and High-Order potentials before using the Back-Mean field method.

5.2. Unsupervised Training

In the absence of annotated training data, inter-view consistency and translation invariance still provide precious a-priori information, which can be leveraged to train our model in an unsupervised way.

Let us assume that the background-subtracting part of the network, which computes f_b, the MLP introduced in Section 3.2.2, is reasonably initialized. In practice, it is easy to do either by training it on a segmentation dataset or by relying on simple background subtraction to compute f_b. Then, starting from initial values of the parameters θ, we first compute the Mean-Field approximation of $P(Z;I_0,\theta)$, which gives us a first lower bound of the partition function. We then sample Z from Q and use that to train our potentials separately as if these samples were ground truth-data, using the supervised procedure of Section 5.1. We then iterate this procedure, that is, Mean-Field inference, sampling from Z, and optimizing the potentials sequentially. This can be interpreted as an Expectation-Maximization (EM) [7] procedure to optimize an Expected Lower Bound (ELB) to the partition function $\mathcal{Z}$ of Eq. 8.

6. Implementation Details

Our implementation uses a single VGGNet-16 Network with pre-trained weights. It computes features that will then be used to estimate both unary and pairwise potentials. The features map $\mathcal{F}^c = \mathcal{F}(I^c;\theta_F)$ is obtained by upsampling of the convolutional layers.

Similarly to the classification step in [23], we restrict the Region-Of-Interest pooling layer (ROI) to the features from the last convolutional layer of VGGNet. The output of the ROI is a 3x3x1024 tensor, which is flattened and input to a two layers MLP with ReLU non-linearities. In a similar way as in previous works on segmentation [29], we use a two layers MLP to classify each hyper-column of our dense features map $\mathcal{F}^c = \mathcal{F}(I^c;\theta_F)$ to produce segmentation f_b and Gaussian Class f_h probabilities.

We use $M = 8$ modes for Multi-Modal Gaussian distribution of Eq. 6 for all our experiments and we have not assessed the impact of this choice on the performance. Besides, our kernel defining the pairwise potentials of Eq. 10 takes an arbitrary uniform constant value. For unsupervised training, we use a fixed number of 6 EM iterations, which we empirically found to be enough, as illustrated in the supplementary material.

Finally, all our pipeline is implemented end-to-end using standard differentiable operations from the Theano Deep-Learning library [24]. For Mean-Field inference, we use a fixed number of iterations (30) and step size (0.01).

7. Evaluation

7.1. Datasets, Metrics, and Baselines

We introduce here the datasets we used for our experiments, the metrics we relied on to evaluate performance, and the baselines to which we compared our approach.

Datasets.

- **ETHZ**. [8] It was acquired using 7 cameras to film the dense flow of students in front of the ETHZ main building in Zürich for two hours. It comprises 250 annotated temporal 7-image frames in which up to 30 people can be present at a time. We used 200 of these frames for training and validation and 50 for evaluation. See the image of Fig. 1 for a visualization.
- **EPFL**. The images were acquired at 25 fps on the terrace of an EPFL building in Lausanne using 4 DV cameras. The image of Fig. 3 is one of them. Up to 7 people walk around for about 3 1/2 minutes. As there are only 80 annotated frames, we used them all for evaluation purposes and relied either on pre-trained models or unsupervised training.
- **PETS**. The standard **PETS 2009** (PETS S2L1) is widely used for monocular and multi-camera detection. It contains 750 annotated images and was acquired from 7 cameras. It is a simple dataset in the sense that it is not very crowded, but the calibration is inaccurate and the image quality low.

Metrics. Recall from Section 4, that our algorithms produces Probabilistic Occupancy Maps, such as the ones of Fig. 3. They are probabilities of presence of people at ground locations and are very peaky. We therefore simply label locations where the probability of presence is greater than 0.5 as being occupied and will refer to these as *detections*, without any need for Non-Maximum suppression. We compute false positive (FP), false negative (FN) and true positives (TP) by assigning detections to ground truth using Hungarian matching. Since we operate in the ground plane, we impose that a detection can be assigned to a ground truth annotation only if they are less than a distance r away. Given FP, FN and TP, we can evaluate:

- **Multiple Object Detection Accuracy (MODA)** which we will plot as a function of r, and the **Multiple Object Detection Precision (MODP)** [14].
- **Precision-Recall**. Precision and Recall are taken to be TP/(TP + FN) and TP/(TP+FP) respectively.

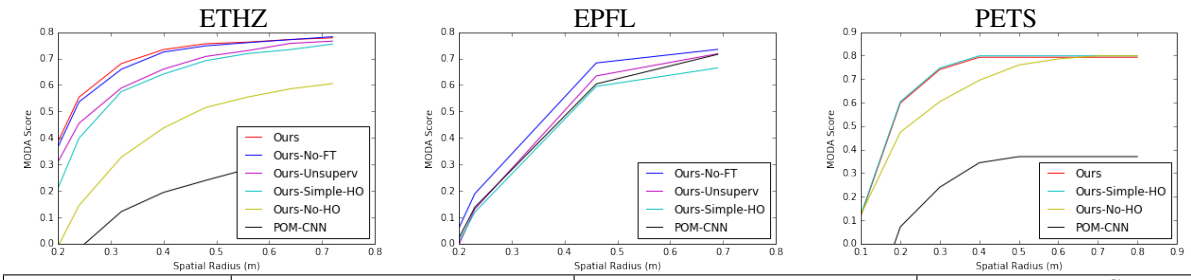
We will report MODP, Precision, and Recall for $r = 0.5$, which roughly corresponds to the width of a human body. Note that these metrics are unforgiving of projection errors because we measure distances in the ground plane, which would not be the case if we evaluated overlap in the image plane as is often done in the monocular case. Nevertheless, we believe them to be the metrics for a multi-camera system that computes the 3D location of people.

Baselines and Variants of our Method. We implemented the following two baselines.

- **POM-CNN**. The multi-camera detector [12] described in Section 2.2 takes background subtraction images as its input. In its original implementation, they were obtained using traditional algorithms [30, 20]. For a fair comparison reflecting the progress that has occurred since then, we use the same CNN-based segmentor as the one use to segment the background, that is $f_{\mathrm{b}}(\mathcal{F}_k^c;\theta_b)_0$ from Eq. 6.
- **RCNN-2D/3D**. The recent work of [27] proposes a MCMT tracking framework that relies on a powerful CNN for detection purposes [23], as discussed in Section 2.2. Since the code of [27] is not publicly available, we reimplemented their detection methodology as faithfully as possible but *without* the tracking component for a fair comparison with our approach that operates on images acquired at the same time. Specifically, we run the 2D detector [23] on each image. We then project the bottom of the 2D bounding box onto the ground reference frame as in [27] to get 3D ground coordinates. Finally, we cluster all the detections from all the cameras using 3D proximity to produce the final set of detections.

To gauge the influence of the different components or our approach, we compared these baselines against the following variants of our method.

- **Ours**. Our method with all three terms in the CRF model turned on, as described in Section 3.3, and fine tuned end-to-end through back Mean-Field, as described in Section 5.1.
- **Ours-No-FT**. **Ours** without the final fine-tuning.
- **Ours-Unsuperv**. Same as **Ours-No-FT** but the training is done without ground truth annotations, as described in Section 5.2.
- **Ours-Simple-HO** : We replace the full High-Order term of Section 3.2 with the simplified one that approximates the one of [12], as described at the beginning of that section.
- **Ours-No-HO**. We remove the High-Order term of Section 3.2 altogether.



	ETHZ		EPFL		PETS	
Method	**Precision / Recall**	**MODP**	**Precision / Recall**	**MODP**	**Precision / Recall**	**MODP**
Ours	**95 / 80%**	**53.8%**	-	-	-	-
Ours-No-FT	93 / 80%	53.4%	**88 / 82%**	**48.3%**	**93 / 87%**	**60.4%**
Ours-Unsuperv	86 / 80%	49.8%	80 / 85%	47.5%	-	-
Ours-Simple-HO	87 / 70%	47.5%	85 / 75%	43.2%	93 / 87%	60.4%
Ours-No-HO	84 / 55%	34.4%	37 / 68%	23.3%	93 / 81%	55.2%
POM-CNN	75 / 55%	30.5%	80 / 78%	45.9%	90 / 86%	42.9%
RCNN-2D/3D	68 / 43%	18.4%	39 / 50%	21.6%	50 / 63%	27.6%

Figure 4. Results on our three test datasets. **Top row.** MODA scores for the different methods as function of the radius r used to compute it, as discussed in Section 7.1. **Bottom row.** Precision/Recall and MODP for the different methods for $r = 0.5$. Some of the values are absent either due to the bad calibration of the data-set, or missing ground-truth, as explained in Sections 7.1 and 7.2. The numbers we report for the **RCNN-2D/3D** baseline are much lower than those reported in [27] for the method that inspired it, in large part because we evaluate our metrics in the ground plane instead of the image plane and because [27] uses a temporal consistency to improve detections.

7.2. Results

We report our results on our three test datasets in Fig. 4.

ETHZ. **Ours** and **Ours-No-FT** clearly dominate the **RCNN-2D/3D** and **POM-CNN** baselines, with **Ours** slightly outperforming **Ours-No-FT** because of the fine-tuning. Simplifying the high-order term, as in **Ours-Simple-HO**, degrades performance and removing it, as in **Ours-No-HO**, degrades it even more. The methods discussed above rely on supervised training, whereas **Ours-Unsuperv** does not but still outperforms the baselines.

EPFL. Because the images have different statistics than those of **ETHZ**, the unary terms as well as the people detector **RCNN-2D/3D** relies on are affected. And since there is no annotated data for retraining, as discussed above, the performance of **Ours-No-HO** and **RCNN-2D/3D** drop very significantly with respect to those obtained on **ETHZ**. By contrast, the high order terms are immune to this, and both **Ours-No-FT** and **Ours-Unsuperv** hold their performances.

PETS. The ranking of the methods is the same as before except for the fact that **Ours-Simple-HO** does as well as **Ours-No-FT**. This is because the **PETS** dataset is poorly calibrated, which results in inaccurate estimates of the displacement vectors in the generative model of Section 3.2.2. As a result, it does not deliver much of a performance boost and we therefore did not find it meaningful to report results for unsupervised training and fine-tuning of these High-Order potentials.

From Detections to Trajectories. Since our method produces a Probability Occupancy Map for every temporal frame in our image sequences, we can take advantage of a simple-flow based method [6] to enforce temporal consistency and produce complete trajectories. As shown in Fig. 5 this leads to further improvements for all three datasets.

Method	ETHZ	EPFL	PETS
Ours	74.1%	68.2%	79.8%
Ours + [6]	75.2%	76.9%	83.4%

Figure 5. MODA scores for $r = 0.5$ before and after enforcing temporal consistency.

8. Discussion

We introduced a new CNN/CRF pipeline that outperforms the state-of-the art for multi-camera people localization in crowded scenes. It handles occlusion while taking full advantage of the power of a modern CNN and can be trained either in a supervised or unsupervised manner.

A limitation, however, is that the CNN used to compute our unary potentials still operates in each image independently as opposed to pooling very early the information from multiple images and then leveraging the expected appearance consistency across views. In future work, we will explore the multi-camera regression method of [9] to improve unary potentials.

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Output of the Gaussian density discriminative network.

As expected the colors of the learned Gaussian classes in Fig. 3 match those predicted by the Network in Fig. 2. We see that our learning algorithm indeed identified several Gaussian classes which can be interpreted as "Left Head and Shoulders", "Right Head and Shoulders", "Right Flank" etc...

1 Detailed Explanation

This document illustrates the output of our Gaussian discriminative model and the learned Gaussian parameters.
The semantic segmentation of Fig. 1 corresponds to the output of the Background/Foreground segmentation Network. It is used in the "simple" occlusion model to compute

$$P^d(X_k = 0) = f_{\mathrm{b}}(\mathcal{F}_k^c; \theta_b) \ ,$$

and in the "full" model to compute

$$P^d(\vec{X}_k = 0) = f_{\mathrm{b}}(\mathcal{F}_k^c; \theta_b) \ .$$

Figure 1: Semantic segmentation $f_{\mathrm{b}}(\mathcal{F}_k^c; \theta_b)$.

The Network output of Fig. 2 represents the Gaussian class output by the network to compute $P^d(\vec{X}_k | \vec{X}_k \neq 0)$. More precisely, each pixel is colored proportionaly to,

$$f_{\mathrm{h}}(\mathcal{F}_k^c; \theta_{\mathrm{h}})_m (1 - f_{\mathrm{b}}(\mathcal{F}_k^c; \theta_b)) \ ,$$

where each Gaussian class m corresponds to a different RGB color. For convenience of representation, we only display 3 Gaussian classes out of 8 used in total.

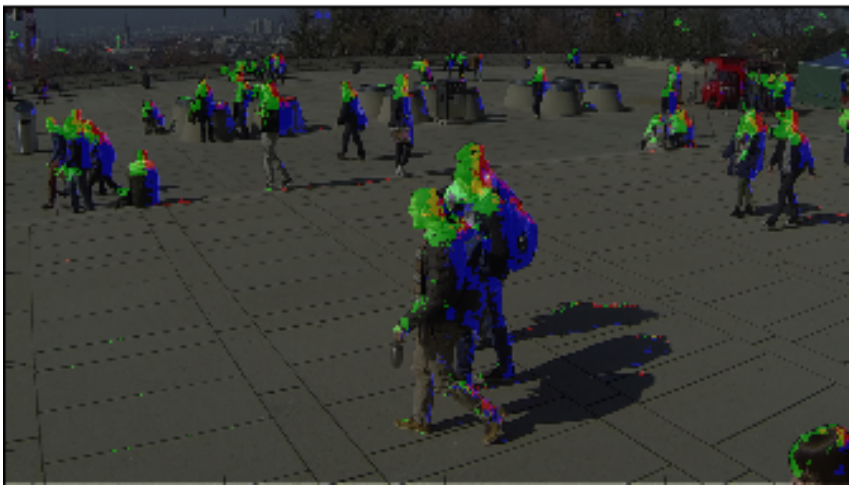

Figure 2: Gaussian Network output $f_{\mathrm{h}}(\mathcal{F}_k^c;\theta_{\mathrm{h}})_m(1 - f_{\mathrm{b}}(\mathcal{F}_k^c;\theta_b))$. We see that the pixels have been correctly identified as belonging to one of the three represented body-part, colored in *Red, Green and Blue*. We see that *Yellow* pixels appear, which correspond to pixels classified both as *Red* and *Green*.

Finally, we propose, in Fig. 3, to visualise the Gaussian parameters learned during training. To do so, we represent a projected bounding box centred in 0 of size $H \times W$. We then use a specific color to highlight the set of pixels which would vote for this bounding-box through each Gaussian element. For instance, the pixels k colored in *red*, where *red* corresponds to Gaussian class $m = 1$, are those such that,

$$\frac{\|x_k - \alpha_{1x}\|^2}{2(H\sigma_{1x})^2} + \frac{\|y_k - \alpha_{1y}\|^2}{2(W\sigma_{1y})^2} \leq 1 \ .$$

This representation has to be put in regards with the discriminative/generative correspondence. Indeed, let us assume that there is a single person in the image, with its single corresponding projected bounding-box in the camera plane. The discriminative model then matches the generative one if the network "colors" the pixels inside the bounding-box as in Fig 3.

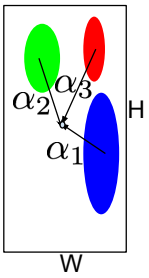

Figure 3: Representation of the learned three learned Gaussians with colors corresponding to the classes of Fig. 2.

2 Results

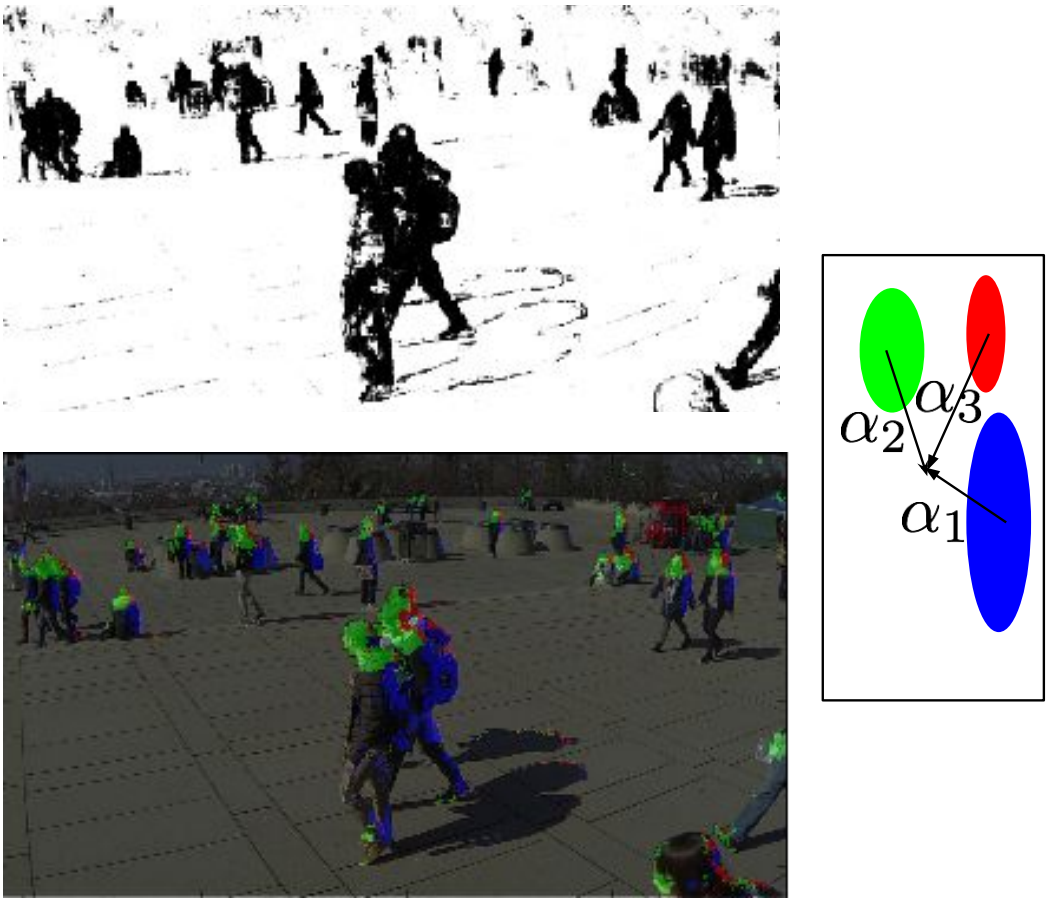

Figure 4: Camera 1. Gaussians 1,2 and 3 represented.

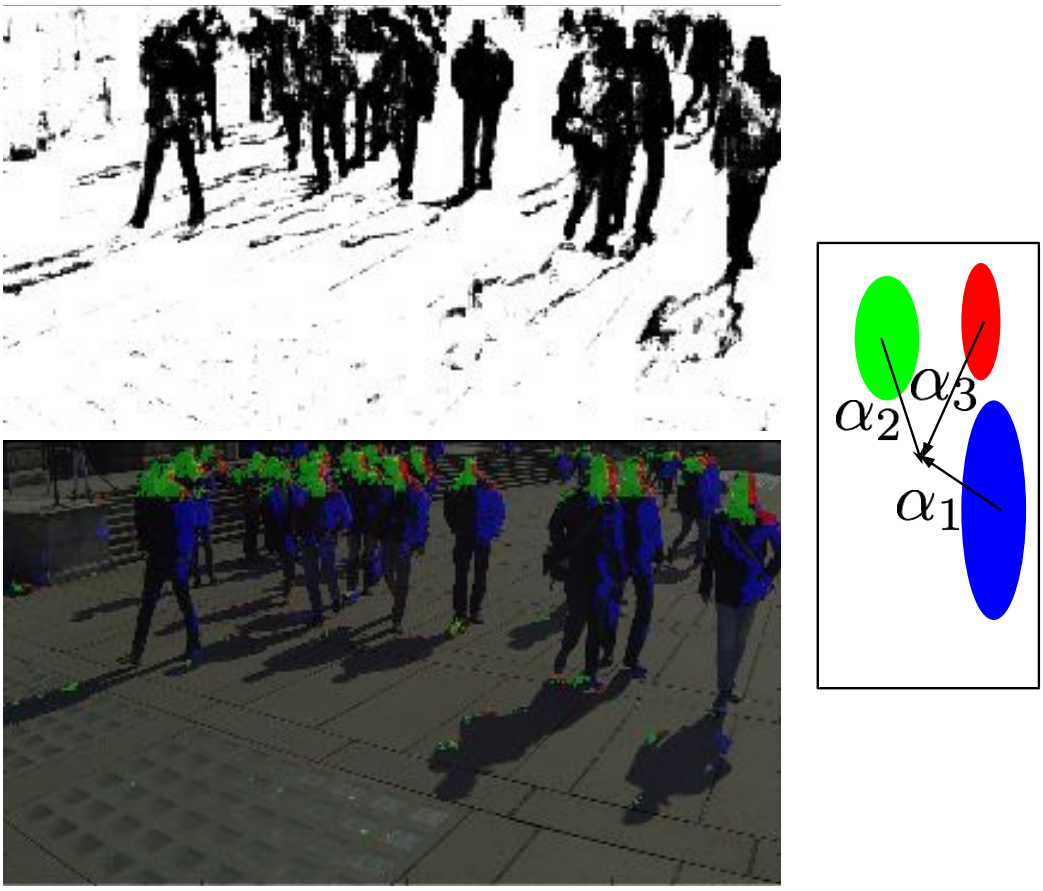

Figure 5: Camera 2. Gaussians 1,2 and 3 represented.

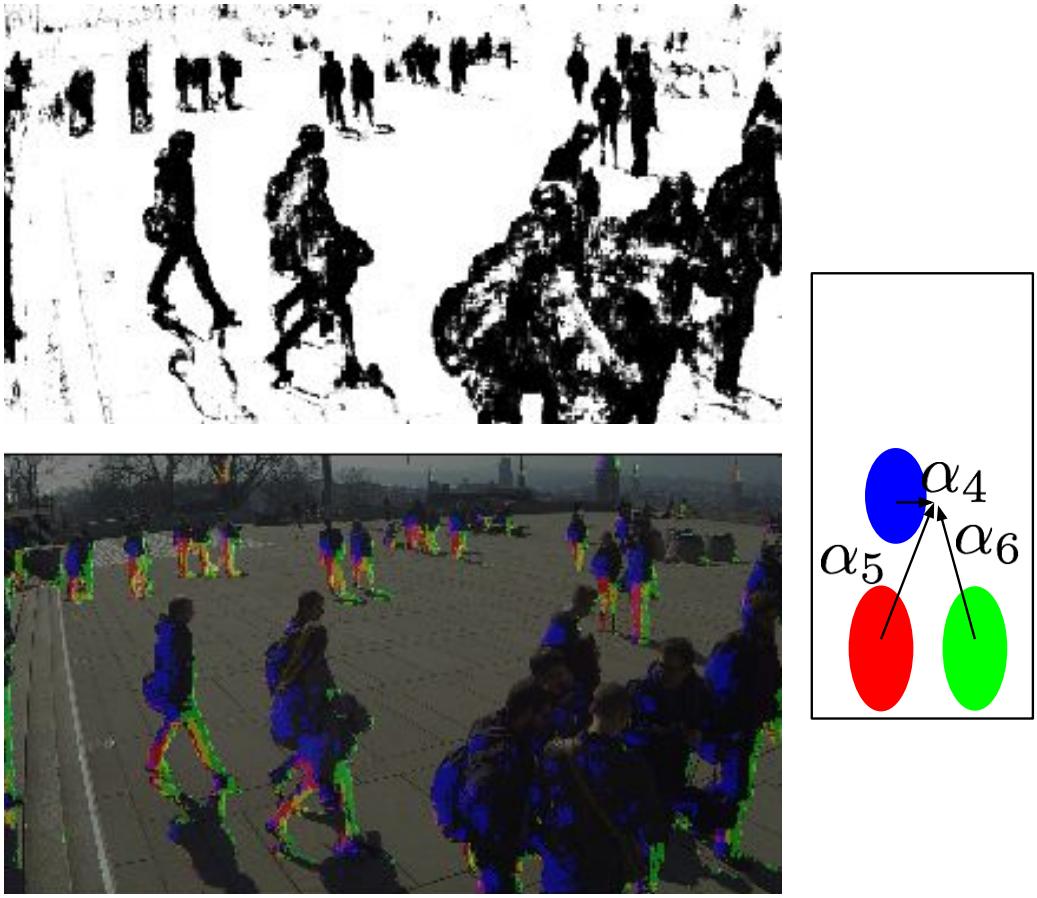

Figure 6: Camera 3. Gaussians 4,5 and 6 represented.

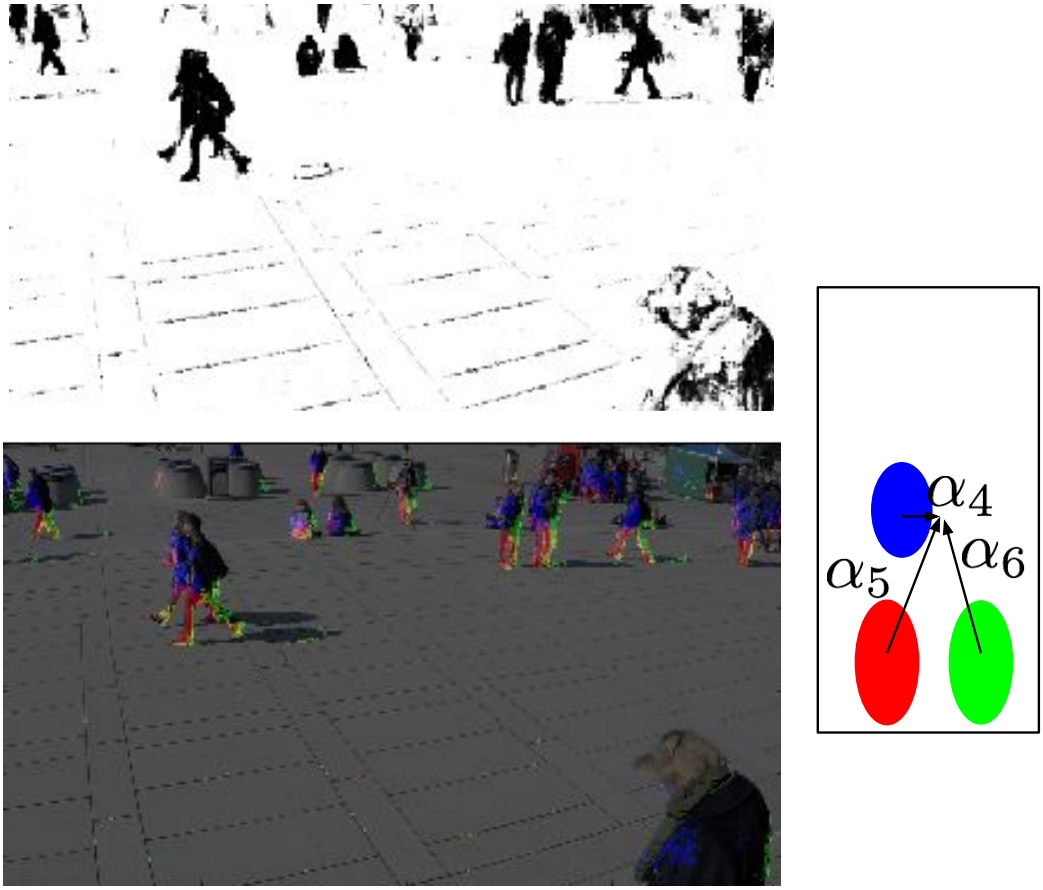

Figure 7: Camera 4. Gaussians 4,5 and 6 represented.

Regression with Gaussian Mixture Networks

In this document, we provide technical details about the discriminative model P^d and its optimisation. Recall that $P^d(\vec{X}_k|\vec{X}_k \neq 0)$ is a Gaussian Mixture probability distribution in $\mathbb{R}^2$, where the weight of each Gaussian is predicted by a Neural Network.

We therefore need to learn the parameters of the probability distribution

$$P^d(\vec{X}_k|\vec{X}_k \neq 0) = \sum_{1 \leq m \leq M} f_{\text{h}}(\mathcal{F}_k^c; \theta_{\text{h}})_m \mathcal{N}(\vec{X}_k - \alpha_m; \sigma_m) \ , \tag{1}$$

namely, the Gaussian parameters α_m and σ_m for each mode index m, and the network parameters θ_{h}.

Following Eq.17 of the main paper, we treat each pixel as an independent data-point $\vec{x}_s$. $S = S(Z^0, \ldots, Z^D)$ denotes the set of those data-points. In this section, we assume that we have access to a label in $\mathbb{R}^2$, for each data-point $\vec{x}_s$. We recall that this label is generated by sampling of the generative model, given *ground truth* detections Z.

The procedure that we use to optimise the following loss derived from Eq.17

$$\mathcal{R}(\theta_{\text{h}}, \alpha, \sigma) = -\sum_{(\vec{x}_s \in S} \log(P^d(\vec{x}_s|\mathcal{F}_{k_s}^{c_s}, \theta_{\text{h}}, \alpha, \sigma)) \ , \tag{2}$$

follows the same principles as the standard Gaussian Mixture regression model via Expectation-Maximization algorithm [1] and is also closely related to the recent Neural Decision Forests [2], which introduce a Network producing a probability distribution in the form of a mixture of Histograms.

Updating the Network Parameters We update the parameters of the network θ_{h} by direct back-propagation and stochastic gradient descent on the objective of Eq. 2.

Updating the Gaussian Parameters Let α^t and σ^t denote the current estimates of the the Gaussian parameters. We derive a closed form update which guarantees that

$$\mathcal{R}(\theta_{\text{h}}, \alpha^{t+1}, \sigma^{t+1}) \leq \mathcal{R}(\theta_{\text{h}}, \alpha^t, \sigma^t) \ . \tag{3}$$

For each data point $\vec{x}_s$, let us introduce the distribution over the mixture elements m,

$$\xi^t(m|\vec{x}_s, \mathcal{F}_{k_s}^{c_s}, \theta_{\text{h}}, \alpha^t, \sigma^t) = \frac{f_{\text{h}}(\mathcal{F}_{k_s}^{c_s}; \theta_{\text{h}})_m \mathcal{N}(\vec{x}_s - \alpha_m^t; \sigma_m^t)}{\sum_{1 \leq m' \leq M} f_{\text{h}}(\mathcal{F}_{k_s}^{c_s}; \theta_{\text{h}})_{m'} \mathcal{N}(\vec{x}_s - \alpha_{m'}^t; \sigma_{m'}^t)} \tag{4}$$

usually called "responsibilities" in the GMM litterature.

We then use the standard variational trick with the auxiliary distribution $\xi^t(m)$ to minimise an upper-bound on $\mathcal{R}(\theta_{\text{h}}, \alpha, \sigma)$, with respect to the parameters α and σ.

$$\begin{aligned}
\mathcal{R}(\theta_{\text{h}}, \alpha, \sigma) &= -\sum_{\vec{x}_s \in S} \log \left(\sum_{1 \leq m \leq M} f_{\text{h}}(\mathcal{F}_{k_s}^{c_s}; \theta_{\text{h}})_m \mathcal{N}(\vec{x}_s - \alpha_m; \sigma_m) \right) \\
&= -\sum_{\vec{x}_s \in S} \log \left(\sum_{1 \leq m \leq M} \xi^t(m|\vec{x}_s) \frac{f_{\text{h}}(\mathcal{F}_{k_s}^{c_s}; \theta_{\text{h}})_m \mathcal{N}(\vec{x}_s - \alpha_m; \sigma_m)}{\xi^t(m|\vec{x}_s)} \right) \\
&\leq -\sum_{\vec{x}_s \in S} \sum_{1 \leq m \leq M} \xi^t(m|\vec{x}_s) \log \left(\frac{f_{\text{h}}(\mathcal{F}_{k_s}^{c_s}; \theta_{\text{h}})_m \mathcal{N}(\vec{x}_s - \alpha_m; \sigma_m)}{\xi^t(m|\vec{x}_s)} \right) \\
&\leq \mathcal{R}(\theta_{\text{h}}, \alpha^t, \sigma^t) - \sum_{\vec{x}_s \in S} \sum_{1 \leq m \leq M} \xi^t(m|\vec{x}_s) \log \left(\frac{\mathcal{N}(\vec{x}_s - \alpha_m; \sigma_m)}{\mathcal{N}(\vec{x}_s - \alpha_m^t; \sigma_m^t)} \right) \qquad (5)
\end{aligned}$$

Minimizing Eq. 5 with respect to α and σ is a convex problem. Assuming that we can find the values achieving the minimum, let us set α^{t+1} and σ^{t+1} to these. Then, from Eq. 5, we obtain

$$\mathcal{R}(\theta_{\text{h}}, \alpha^{t+1}, \sigma^{t+1}) \leq \mathcal{R}(\theta_{\text{h}}, \alpha^t, \sigma^t) \ ,$$

with equality if and only if $\alpha^{t+1} = \alpha^t$ and $\sigma^{t+1} = \sigma^t$.

We therefore need to minimize Eq. 5 with respect to α and σ, which is equivalent to maximizing

$$\sum_{\vec{x}_s \in S} \sum_{1 \leq m \leq M} \xi^t(m|\vec{x}_s) \log \left(\mathcal{N}(\vec{x}_s - \alpha_m; \sigma_m) \right) \ ,$$

with respect to the parameters α and σ. This is done by using the standard optimality conditions for convex problems. We obtain

$$\alpha_m^{t+1} = \frac{\sum_{\vec{x}_s \in S} \xi^t(m|\vec{x}_s) \vec{x}_s}{\sum_{\vec{x}_s \in S} \xi^t(m|\vec{x}_s)} \ , \qquad (6)$$

and,

$$\sigma_m^{t+1} = \frac{\sum_{\vec{x}_s \in S} \xi^t(m|\vec{x}_s) (\vec{x}_s - \alpha_m^{t+1})^2}{\sum_{\vec{x}_s \in S} \xi^t(m|\vec{x}_s)} \ . \qquad (7)$$

Alternating both In practice, we alternate one epoch of stochastic gradient descent optimizing the network parameters θ_{h} with one update of the Gaussian parameters of Eqs. 6 and 7. For memory usage reasons, the sums in Eqs. 6 and 7, have to be split into mini-batches. However the update is done after summation over the whole dataset or a very large number of samples.

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Efficient differentiable implementation of Mean-Field inference for High-Order Potentials.

In this document, we concisely explain how Mean-Fields (MF) inference can be efficiently implemented with the High-Order potentials described in the main paper.

More precisely, for each pixel k and location i, we seek to compute efficiently the approximated natural gradient term

$$\widetilde{\nabla \eta_i}^k = -C^k \left(\mathbb{E}_Q \left[\Delta^k(Z) | Z_i = 1 \right] - \mathbb{E}_Q \left[\Delta^k(Z) | Z_i = 0 \right] \right) , \tag{1}$$

and then sum these terms for every pixel to obtain the approximated natural gradient $\widetilde{\nabla \eta_i}$.

Computing each natural gradient term Recall that $\Delta^k(Z)$ is a function of Z which takes value 0 if one of the "compatible explanations" for pixel k is present and 1 otherwise. Also, recall that we say that an explanation Z_i is compatible if a presence in Z_i gets projected on the camera plane in such a way that it matches the observation at pixel k produced by the discriminative model. Let $\mathcal{C}^k$ denote the list of indices $j \in \{1, \dots, N\}$ such that a presence in Z_j is a compatible explanation for the observation at pixel k.

Let us consider pixel k and location i. If location i is not compatible with pixel k (i.e. $i \notin \mathcal{C}^k$), then the value taken by Z_i has no impact on $\Delta^k(Z)$ and therefore $\widetilde{\nabla \eta_i}^k = 0$.

Let us assume that $i \in \mathcal{C}^k$. Then,

$$\mathbb{E}_Q \left[\Delta^k(Z) | Z_i = 1 \right] = 0$$

and,

$$\begin{aligned} \mathbb{E}_Q \left[\Delta^k(Z) | Z_i = 0 \right] &= \prod_{j \in \mathcal{C}^k / i} (1 - Q(Z_j = 1)) \\ &= \frac{\prod_{j \in \mathcal{C}^k} (1 - Q(Z_j = 1))}{1 - Q(Z_i = 1)} , \end{aligned} \tag{2}$$

where the first equation the fact that the MF distribution Q is fully factorized.

Computing Updates for all variables in two steps Computing the gradient term of Eq. 2 directly would require a large multiplication for each pixel, which would be inefficient. However, we remark that the numerator of Eq. 2, doesn't depend on the chosen i, and its denominator doesn't depend on k. We therefore proceed using the two following steps

- For each pixel k, we compute

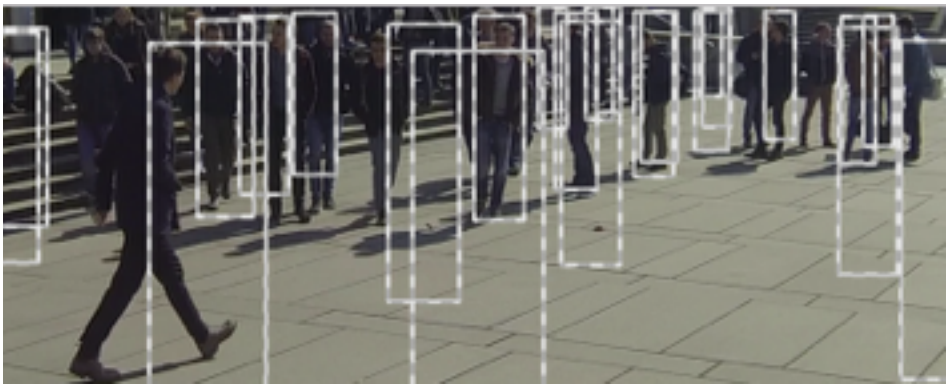
$$\delta_k = \prod_{j \in \mathcal{C}^k} (1 - Q(Z_j = 1)) . \tag{3}$$

- Then, for each variable index i, we compute the sum over all pixels

$$\widetilde{\nabla \eta_i} = \frac{1}{1 - Q(Z_i = 1)} \sum_{k | i \in \mathcal{C}^k} \delta_k \ . \tag{4}$$

Note that these operations are all differentiable with respect to the MF distribution Q and to the parameter C^k, which makes it possible to back-propagate the gradient through the MF iterations.

Furthermore, since the Gaussians were approximated for inference by constant terms, on a rectangular zones, the sum of Eq. 4, can be computed efficiently using integral images.



RCNN-2D/3D : Projecting 2D detections form one image to 3D on the ground plane is prone to large localization errors, especially when

POM : The POM method uses background subtraction (or semantic segmentation in this case) as its sole input. Therefore, in crowded scenes, or when a single camera covers the target, it may fail to correctly reason about the number of people and their location.

DeepOcclusion : Ou method is able to reason directly in 3D, using features extracted by a Deep Network.