Deep learning

4.3. PyTorch modules and batch processing

François Fleuret

https://fleuret.org/ee559/

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Usually `torch.nn.functional` is imported as `F`, and `torch.nn` as `nn`. 
Functions and modules from \texttt{nn} process \texttt{batches} of inputs stored in a tensor whose first dimension indexes them, and produce a corresponding tensor with the same additional dimension.

\textit{E.g.} a fully connected layer $\mathbb{R}^C \rightarrow \mathbb{R}^D$ expects as input a tensor of size $N \times C$ and computes a tensor of size $N \times D$, where $N$ is the number of samples and can vary from a call to another. We come back to this in a second.
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takes a tensor of any size as input, applies ReLU on each value to produce a result tensor of same size.
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```python
>>> x
torch([[ 0.8008, -0.2586, 0.5019, -0.2002, -0.7416],
       [ 0.0557, 0.6046, 0.0864, -0.5929, 1.2606]])
>>> F.relu(x)
torch([[ 0.8008, 0.0000, 0.5019, 0.0000, 0.0000],
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```

`inplace` indicates if the operation should modify the argument itself. This may be desirable to reduce the memory footprint of the processing.
The module

\texttt{nn.Linear(in\_features, out\_features, bias=True)}

implements a $\mathbb{R}^C \rightarrow \mathbb{R}^D$ fully-connected layer. It takes as input a tensor of size $N \times C$ and produce a tensor of size $N \times D$. 
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```python
>>> f = nn.Linear(in_features = 10, out_features = 4)
>>> for n, p in f.named_parameters(): print(n, p.size())
...
weight torch.Size([4, 10])
bias torch.Size([4])
```

```python
>>> x = torch.empty(523, 10).normal_()
>>> y = f(x)
>>> y.size()
torch.Size([523, 4])
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The module

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The weights and biases are automatically randomized at creation. We will come back to that later.
The module

\texttt{nn.MSELoss()}

implements the Mean Square Error loss: the sum of the component-wise
squared difference, \textit{divided by the total number of components in the tensors}. 

\begin{verbatim}
>>> f = nn.MSELoss()
>>> x = torch.tensor([3.])
>>> y = torch.tensor([0.])
>>> f(x, y)
tensor(9.)

>>> x = torch.tensor([[3., 0., 0., 0.]])
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```

The first parameter of a loss is traditionally called the \texttt{input} and the second the \texttt{target}. These two quantities may be of different dimensions or even types for some losses (\textit{e.g.} for classification).
Criteria do not accept a target with `requires_grad` to `True`.

```python
>>> import torch
>>> f = nn.MSELoss()
>>> x = torch.tensor([ 3., 2. ]).requires_grad_()
>>> y = torch.tensor([ 0., -2. ]).requires_grad_()
>>> f(x, y)
Traceback (most recent call last):
  .../
AssertionError: nn criterions don't compute the gradient w.r.t.
targets - please mark these tensors as not requiring gradients
```
Batch processing
Functions and modules from \texttt{nn} process samples by batches. This is motivated by the computational speed-up it induces.

Training a large network on CIFAR10:

<table>
<thead>
<tr>
<th>Batch size</th>
<th>Time per epoch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4h22min</td>
</tr>
<tr>
<td>64</td>
<td>4min50s</td>
</tr>
</tbody>
</table>

speed up of \( \times 54 \).
To evaluate a module on a sample, both the module’s parameters and the sample have to be first copied into cache memory, which is fast but small.

For any model of reasonable size, only a fraction of its parameters can be kept in cache, so a module’s parameters have to be copied there every time they are used.
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Memory transfers are slower than computation. Batch processing cuts down to one copy of the parameters to the cache per batch.

It also cuts down the use of Python loops, which are awfully slow.
Consider a model composed of three modules

\[ f = f_3 \circ f_2 \circ f_1, \]

and we want to compute \( f(x_1), f(x_2), f(x_3) \).
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Processing samples one by one:

- Copying the \( x_n \)s to cache memory
- Copying the \( f_d \)s’ parameters to cache memory
- Computing a \( f_d(\cdot) \)

Time
Consider a model composed of three modules

\[ f = f_3 \circ f_2 \circ f_1, \]

and we want to compute \( f(x_1), f(x_2), f(x_3) \).

Processing samples one by one:

Batch processing:
With

def timing(x, w, batch = False, nb = 101):
    t = torch.zeros(nb)

    for u in range(nb):
        t0 = time.perf_counter()
        if batch:
            y = x.mm(w.t())
        else:
            y = torch.empty(x.size(0), w.size(0))
            for k in range(y.size(0)): y[k] = w.mv(x[k])
            y.is_cuda and torch.cuda.synchronize()
        t[u] = time.perf_counter() - t0

    return t.median().item()
x = torch.empty(2500, 1000).normal_()
w = torch.empty(1500, 1000).normal_()
print('Batch-processing speed-up on CPU %.1f' %
     (timing(x, w, batch = False) / timing(x, w, batch = True)))

x, w = x.to('cuda'), w.to('cuda')
print('Batch-processing speed-up on GPU %.1f' %
     (timing(x, w, batch = False) / timing(x, w, batch = True)))

prints

Batch-processing speed-up on CPU 4.6
Batch-processing speed-up on GPU 144.4
Formally, we have to revisit a bit some expressions we saw previously for fully connected layers. We had

$$\forall l, n, \ w^{(l)} \in \mathbb{R}^{d_l \times d_{l-1}}, \ x_n^{(l-1)} \in \mathbb{R}^{d_{l-1}}, \ s_n^{(l)} = w^{(l)} x_n^{(l-1)}.$$
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From now on, we will use row vectors, so that we can represent a series of samples as a 2d array with the first index being the sample’s index.

$$x = \begin{pmatrix} x_{1,1} & \cdots & x_{1,D} \\ \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,D} \end{pmatrix} = \begin{pmatrix} (x_1)^T \\ \vdots \\ (x_N)^T \end{pmatrix},$$

which is an element of $\mathbb{R}^{N \times D}$. 
To make all sample row vectors and apply a linear operator, we want
\[ \forall n, \ s_n^{(l)} = \left( w^{(l)} \left( x_n^{(l-1)} \right)^\top \right)^\top = x_n^{(l-1)} \left( w^{(l)} \right)^\top \]
which gives a tensorial expression for the full batch
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which gives a tensorial expression for the full batch

$$s^{(l)} = x^{(l-1)} \left( w^{(l)} \right)^\top.$$
Similarly for the backward pass of a linear layer we get

\[
\begin{bmatrix}
\frac{\partial \mathcal{L}}{\partial w^{(l)}}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \mathcal{L}}{\partial s^{(l)}}
\end{bmatrix}^\top x^{(l-1)},
\]

and

\[
\begin{bmatrix}
\frac{\partial \mathcal{L}}{\partial x^{(l)}}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \ell}{\partial s^{(l+1)}}
\end{bmatrix} w^{(l+1)}.
\]
The end