EE-559 – Deep learning

4.3. PyTorch modules and batch processing

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Usually `torch.nn.functional` is imported as `F`, and `torch.nn` as `nn`. 
Functions and modules from `nn` process **batches** of inputs stored in a tensor whose first dimension indexes them, and produce a corresponding tensor with the same additional dimension.

*E.g.* a fully connected layer $\mathbb{R}^C \rightarrow \mathbb{R}^D$ expects as input a tensor of size $N \times C$ and computes a tensor of size $N \times D$, where $N$ is the number of samples and can vary from a call to another. We come back to this in a second.
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>>> x
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`inplace` indicates if the operation should modify the argument itself. This may be desirable to reduce the memory footprint of the processing.
The module

```
nn.Linear(in_features, out_features, bias=True)
```

implements a $\mathbb{R}^C \rightarrow \mathbb{R}^D$ fully-connected layer. It takes as input a tensor of size $N \times C$ and produces a tensor of size $N \times D$. 

```bash
>>> f = nn.Linear(in_features = 10, out_features = 4)
>>> for n, p in f.named_parameters(): print(n, p.size())
... weight torch.Size([4, 10])
... bias torch.Size([4])
>>> x = torch.empty(523, 10) .normal_()
>>> y = f(x)
>>> y.size()
torch.Size([523, 4])
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⚠️ The weights and biases are automatically randomized at creation. We will come back to that later.
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The first parameter of a loss is traditionally called the \texttt{input} and the second the \texttt{target}. These two quantities may be of different dimensions or even types for some losses (\textit{e.g.} for classification).
Criteria do not accept a target with `requires_grad` to `True`.

```python
>>> import torch
>>> f = nn.MSELoss()
>>> x = torch.tensor([3., 2.]).requires_grad_()
>>> y = torch.tensor([0., -2.]).requires_grad_()
>>> f(x, y)
Traceback (most recent call last):
  .../
AssertionError: nn criterions don't compute the gradient w.r.t. targets - please mark these tensors as not requiring gradients
```
Batch processing
Functions and modules from `nn` process samples by batches. This is motivated by the computational speed-up it induces.

Training a large network on CIFAR10:

<table>
<thead>
<tr>
<th>Batch size</th>
<th>Time per epoch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4h22min</td>
</tr>
<tr>
<td>64</td>
<td>4min50s</td>
</tr>
</tbody>
</table>

speed up of $\times 54$. 
To evaluate a module on a sample, both the module’s parameters and the sample have to be first copied into cache memory, which is fast but small.

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Memory transfers are slower than computation. Batch processing cuts down to one copy of the parameters to the cache per batch.

It also cuts down the use of Python loops, which are awfully slow.
Consider a model composed of three modules

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- Copying the \( x_n \)'s to cache memory
- Copying the \( f_d \)'s' parameters to cache memory
- Computing a \( f_d(.) \)

Processing samples one by one:
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Processing samples one by one:

Batch processing:
With

def timing(x, w, batch = False, nb = 101):
    t = torch.zeros(nb)

    for u in range(nb):
        t0 = time.perf_counter()
        if batch:
            y = x.mm(w.t())
        else:
            y = torch.empty(x.size(0), w.size(0))
            for k in range(y.size(0)): y[k] = w.mv(x[k])
        y.is_cuda and torch.cuda.synchronize()
        t[u] = time.perf_counter() - t0

    return t.median().item()
x = torch.empty(2500, 1000).normal_()
w = torch.empty(1500, 1000).normal_()
print('Batch-processing speed-up on CPU %.1f' %
    (timing(x, w, batch = False) / timing(x, w, batch = True)))

x, w = x.to('cuda'), w.to('cuda')
print('Batch-processing speed-up on GPU %.1f' %
    (timing(x, w, batch = False) / timing(x, w, batch = True)))

prints

Batch-processing speed-up on CPU 4.6
Batch-processing speed-up on GPU 144.4
Formally, we have to revisit a bit some expressions we saw previously for fully connected layers. We had

\[ \forall l, n, \ w^{(l)} \in \mathbb{R}^{d_l \times d_{l-1}}, \ x^{(l-1)}_n \in \mathbb{R}^{d_{l-1}}, \ s^{(l)}_n = w^{(l)} \ x^{(l-1)}_n. \]
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From now on, we will use row vectors, so that we can represent a series of samples as a 2d array with the first index being the sample's index.

\[
\begin{pmatrix}
  x_{1,1} & \ldots & x_{1,D} \\
  \vdots & \ddots & \vdots \\
  x_{N,1} & \ldots & x_{N,D}
\end{pmatrix}
= \begin{pmatrix}
  (x_1)^T \\
  \vdots \\
  (x_N)^T
\end{pmatrix},
\]

which is an element of \( \mathbb{R}^{N \times D} \).
To make all sample row vectors and apply a linear operator, we want

$$\forall n, \ s_n^{(l)} = \left( w^{(l)} \left( x_n^{(l-1)} \right)^T \right)^T = x_n^{(l-1)} \left( w^{(l)} \right)^T$$

which gives a tensorial expression for the full batch

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And in `torch/nn/functional.py`

```python
def linear(input, weight, bias=None):
    if input.dim() == 2 and bias is not None:
        # fused op is marginally faster
        return torch.addmm(bias, input, weight.t())

    output = input.matmul(weight.t())
    if bias is not None:
        output += bias
    return output
```

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Similarly for the backward pass of a linear layer we get

\[
\left[ \frac{\partial \mathcal{L}}{\partial w^{(l)}} \right] = \left[ \frac{\partial \mathcal{L}}{\partial s^{(l)}} \right]^T x^{(l-1)},
\]

and

\[
\left[ \frac{\partial \mathcal{L}}{\partial x^{(l)}} \right] = \left[ \frac{\partial \ell}{\partial s^{(l+1)}} \right] w^{(l+1)}.
\]
The end