EE-559 – Deep learning

9.1. Transposed convolutions

François Fleuret

https://fleuret.org/ee559/

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Generative processes that consist of optimizing the input rely on back-propagation to expend the signal from a low-dimension representation to the high-dimension signal space.

The same can be done in the forward pass with **transposed convolution layers** whose forward operation corresponds to a convolution layer's backward pass.

Consider a 1d convolution with a kernel κ

$$
\begin{aligned}
y_i &= (x \circledast \kappa)_i \\
&= \sum_a x_{i+a-1}\, \kappa_a \\
&= \sum_u x_u\, \kappa_{u-i+1}.
\end{aligned}
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We get

$$\begin{aligned} \left[\frac{\partial \ell}{\partial x}\right]_u &= \frac{\partial \ell}{\partial x_u} \\ &= \sum_i \frac{\partial \ell}{\partial y_i} \frac{\partial y_i}{\partial x_u} \\ &= \sum_i \frac{\partial \ell}{\partial y_i} \kappa_{u-i+1}. \end{aligned}$$

which looks a lot like a standard convolution layer, except that the kernel coefficients are visited in reverse order.

This is actually the standard convolution operator from signal processing. If $*$ denotes this operation, we have

$$(x * \kappa)_i = \sum_a x_a \, \kappa_{i-a+1}.$$

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Coming back to the backward pass of the convolution layer, if

$$y = x \circledast \kappa$$

then

$$\left[\frac{\partial \ell}{\partial x}\right] = \left[\frac{\partial \ell}{\partial y}\right] * \kappa.$$

In the deep-learning field, since it corresponds to transposing the weight matrix of the equivalent fully-connected layer, it is called a **transposed convolution**.

$$
\begin{pmatrix}
\kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 & 0 \\
0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 \\
0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 \\
0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 \\
0 & 0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3
\end{pmatrix}^T
=
\begin{pmatrix}
\kappa_1 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_1 & 0 & 0 & 0 \\
\kappa_3 & \kappa_2 & \kappa_1 & 0 & 0 \\
0 & \kappa_3 & \kappa_2 & \kappa_1 & 0 \\
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0 & 0 & 0 & \kappa_3 & \kappa_2 \\
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\end{pmatrix}
$$

While a convolution can be seen as a series of inner products, a transposed convolution can be seen as a weighted sum of translated kernels.

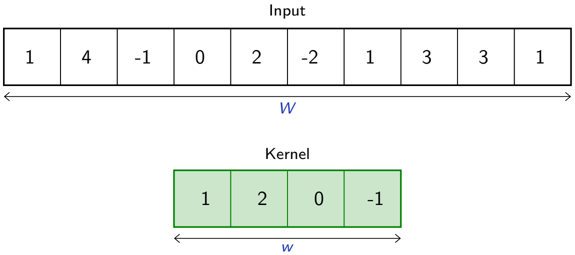
Convolution layer

Input

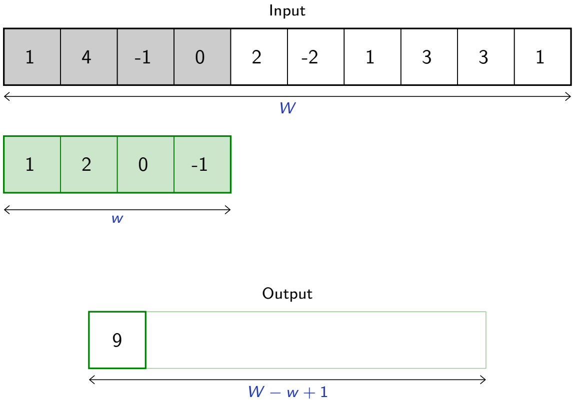
1	4	-1	0	2	-2	1	3	3	1

W

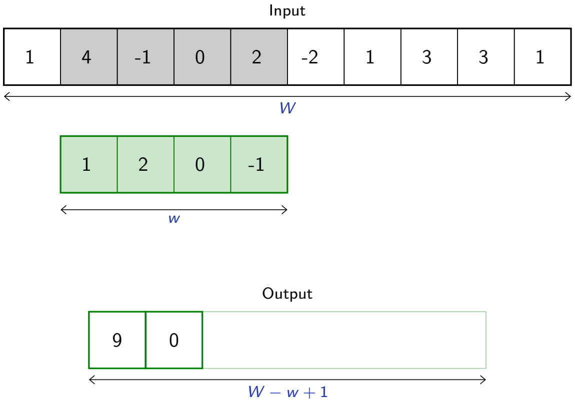
Convolution layer



Convolution layer



Convolution layer



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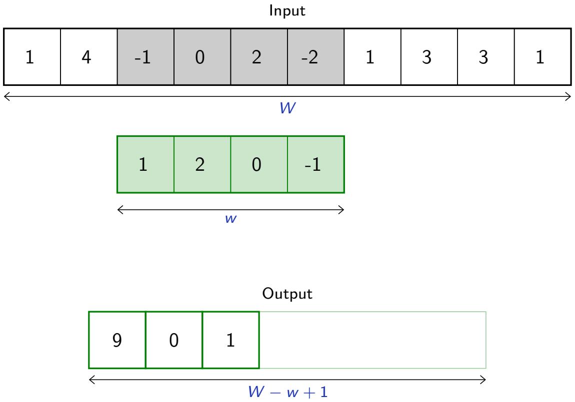

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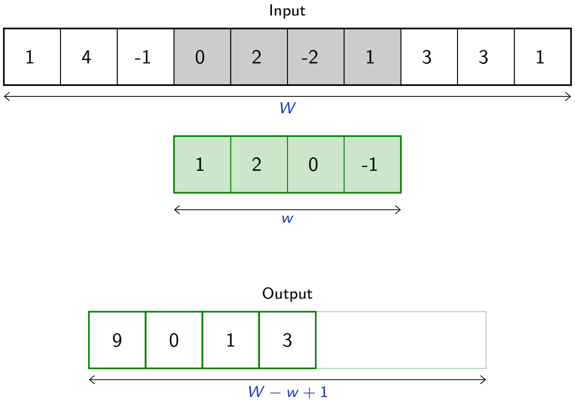

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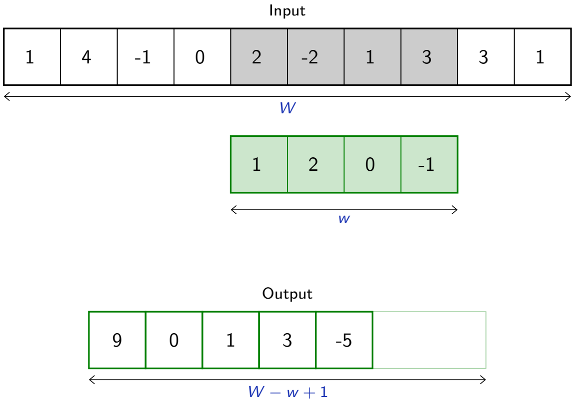

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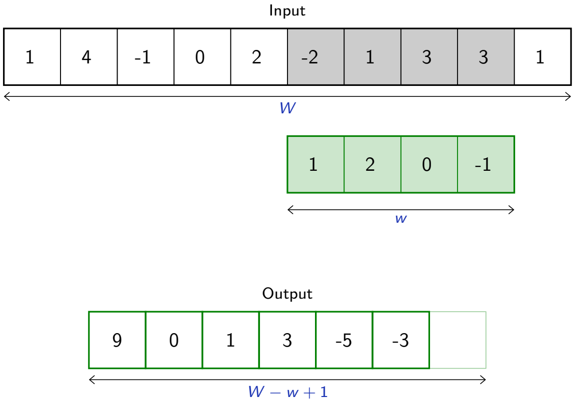

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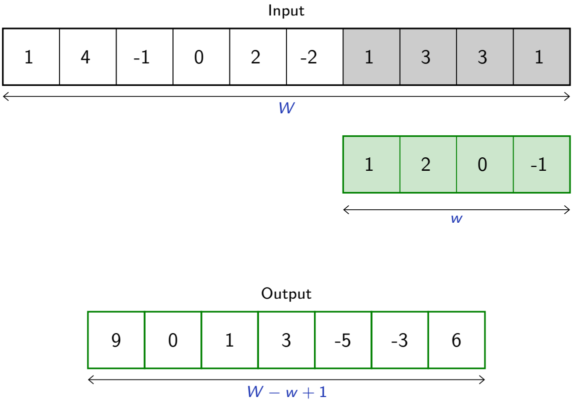

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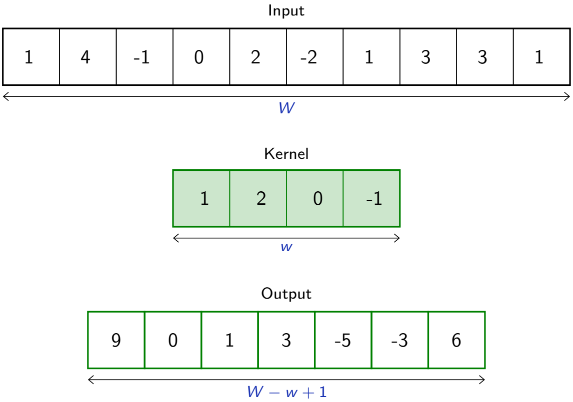
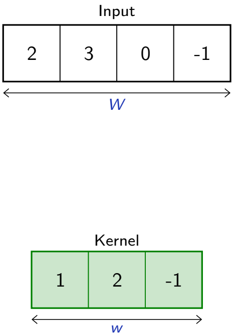


Transposed convolution layer



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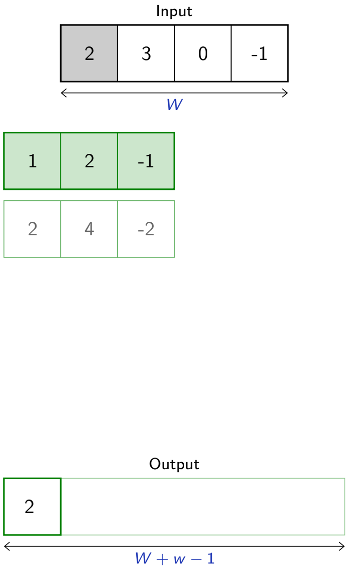

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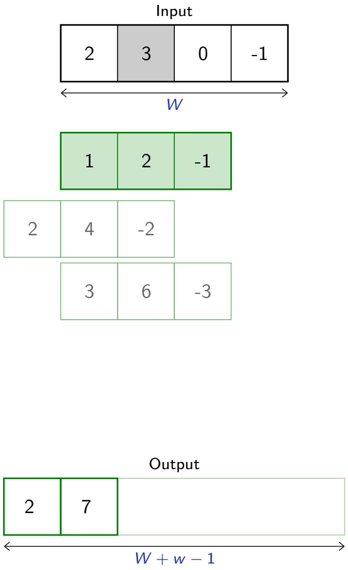

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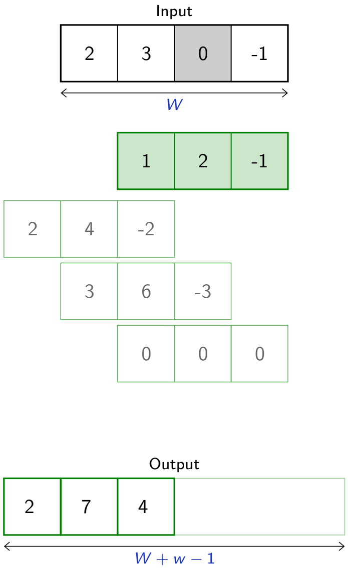

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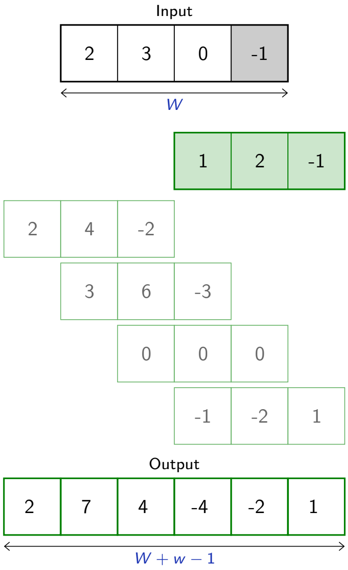

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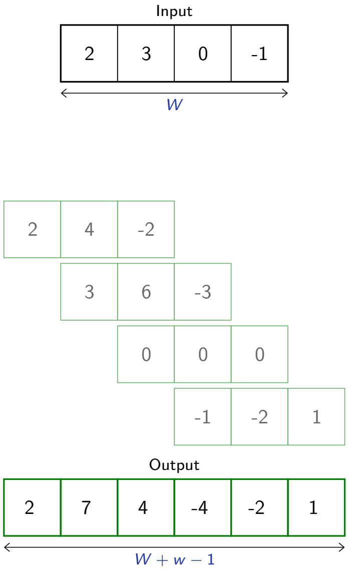

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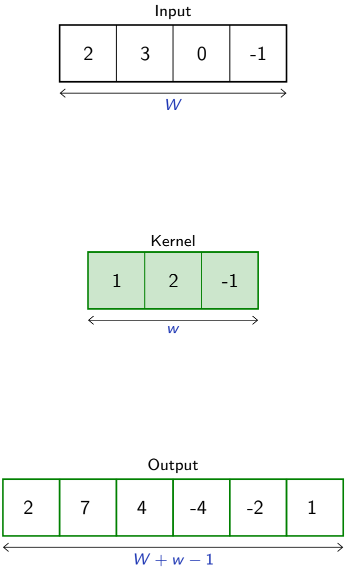
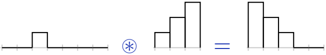


`torch.nn.functional.conv_transpose1d` implements the operation we just described. It takes as input a batch of multi-channel samples, and produces a batch of multi-channel samples.

```
>>> x = torch.tensor([[[0., 0., 1., 0., 0., 0., 0.]]])
>>> k = torch.tensor([[[1., 2., 3.]]])
>>> F.conv1d(x, k)
tensor([[[ 3.,  2.,  1.,  0.,  0.]]])
```



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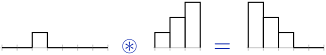
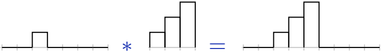


```
>>> F.conv_transpose1d(x, k)
tensor([[[ 0.,  0.,  1.,  2.,  3.,  0.,  0.,  0.,  0.]]])
```



The class `torch.nn.ConvTranspose1d` embeds that operation into a `torch.nn.Module`.

```
>>> x = torch.tensor([[[ 2., 3., 0., -1.]]])
>>> m = nn.ConvTranspose1d(1, 1, kernel_size=3)
>>> m.bias.data.zero_()
tensor([0.])
>>> m.weight.data.copy_(torch.tensor([ 1, 2, -1 ]))
tensor([[[ 1.,  2., -1.]]])
>>> y = m(x)
>>> y
tensor([[[ 2.,  7.,  4., -4., -2.,  1.]]], grad_fn=<SqueezeBackward1>)
```

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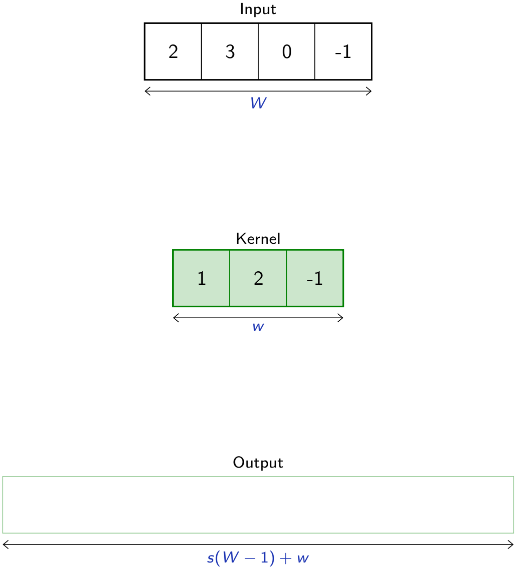
They also have a `stride` and `padding` parameters, however, due to the relation between convolutions and transposed convolutions:

⚠ While for convolutions `stride` and `padding` are defined in the input map, for transposed convolutions these parameters are defined in the output map, and the latter modulates a cropping operation.

Transposed convolution layer (stride $= 2$)



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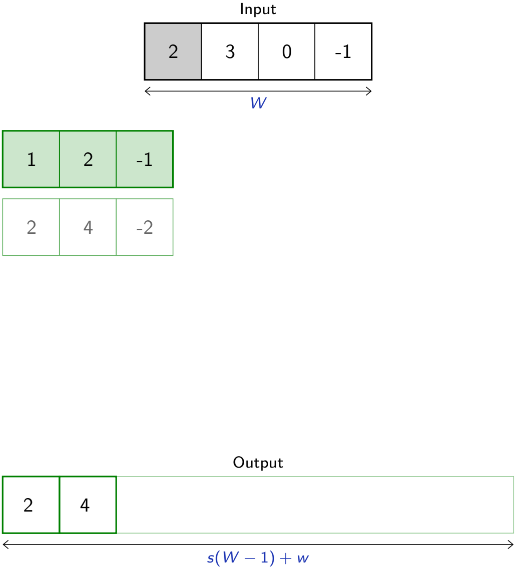
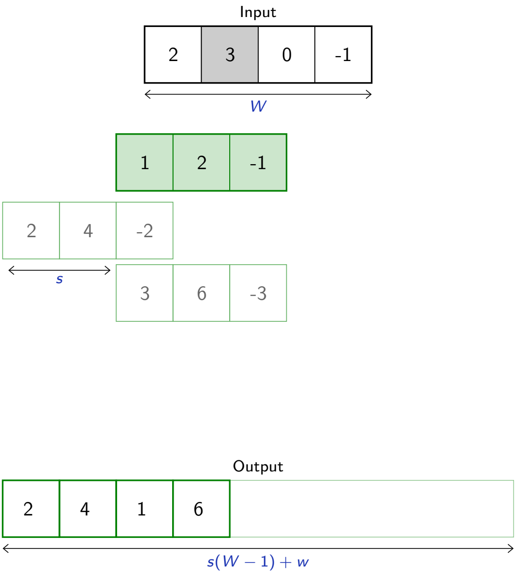


Transposed convolution layer (stride = 2)



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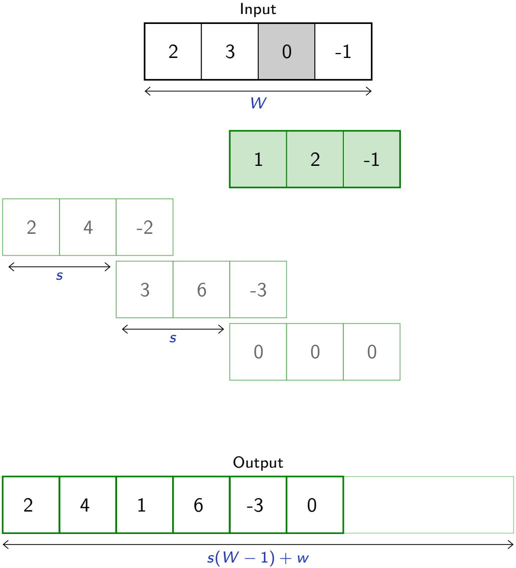

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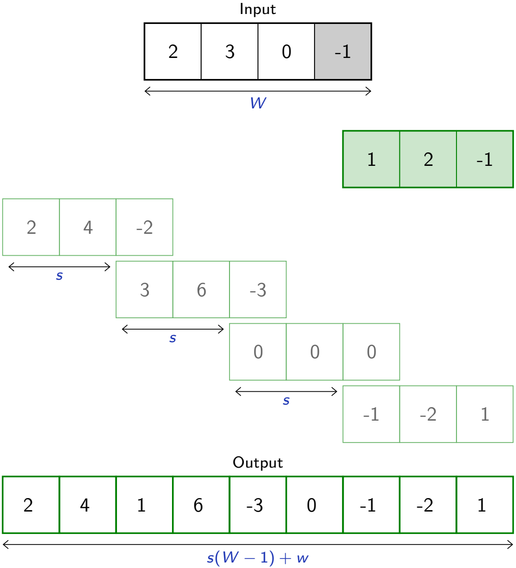

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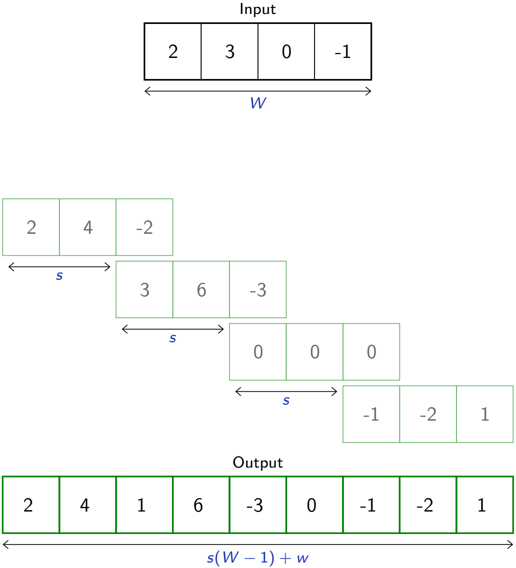

Transposed convolution layer (stride $= 2$)

Input

2	3	0	-1

W

Kernel

1	2	-1

w

Output

2	4	1	6	-3	0	-1	-2	1

$s(W-1)+w$

The composition of a convolution and a transposed convolution of same parameters keep the signal size [roughly] unchanged.

⚠ A convolution with a stride greater than one may ignore parts of the signal. Its composition with the corresponding transposed convolution generates a map **of the size of the observed area**.

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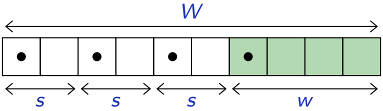
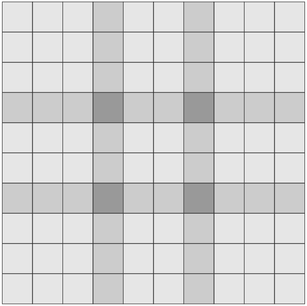
For instance, a 1d convolution of kernel size w and stride s composed with the transposed convolution of same parameters maintains the signal size W, only if

$$\exists q \in \mathbb{N},\ W = w + s\,q.$$



It has been observed that transposed convolutions may create some grid-structure artifacts, since generated pixels are not all covered similarly.

For instance with a 4×4 kernel and stride 3



An alternative is to use an analytic up-scaling, implemented in the PyTorch modules `nn.Upsample`.

```
>>> x = torch.tensor([[[[ 1., 2. ], [ 3., 4. ]]]])
>>> b = nn.Upsample(scale_factor = 3, mode = 'bilinear')
>>> b(x)
tensor([[[[ 1.0000,  1.0000,  1.3333,  1.6667,  2.0000,  2.0000],
          [ 1.0000,  1.0000,  1.3333,  1.6667,  2.0000,  2.0000],
          [ 1.6667,  1.6667,  2.0000,  2.3333,  2.6667,  2.6667],
          [ 2.3333,  2.3333,  2.6667,  3.0000,  3.3333,  3.3333],
          [ 3.0000,  3.0000,  3.3333,  3.6667,  4.0000,  4.0000],
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          [ 1.6667,  1.6667,  2.0000,  2.3333,  2.6667,  2.6667],
          [ 2.3333,  2.3333,  2.6667,  3.0000,  3.3333,  3.3333],
          [ 3.0000,  3.0000,  3.3333,  3.6667,  4.0000,  4.0000],
          [ 3.0000,  3.0000,  3.3333,  3.6667,  4.0000,  4.0000]]]])

>>> u = nn.Upsample(scale_factor = 3, mode = 'nearest')
>>> u(x)
tensor([[[[ 1.,  1.,  1.,  2.,  2.,  2.],
          [ 1.,  1.,  1.,  2.,  2.,  2.],
          [ 1.,  1.,  1.,  2.,  2.,  2.],
          [ 3.,  3.,  3.,  4.,  4.,  4.],
          [ 3.,  3.,  3.,  4.,  4.,  4.],
          [ 3.,  3.,  3.,  4.,  4.,  4.]]]])
```

Such module is usually combined with a convolution to learn local corrections to undesirable artifacts of the up-scaling.

In practice, a transposed convolution such as

```
nn.ConvTranspose2d(nic, noc,
                   kernel_size = 3, stride = 2,
                   padding = 1, output_padding = 1),
```

can be replaced by

```
nn.Upsample(scale_factor = 2, mode = 'bilinear')
nn.Conv2d(nic, noc, kernel_size = 3, padding = 1)
```

The end