Deep learning

5.7. Writing an autograd function

François Fleuret

https://fleuret.org/dlc/
We have seen how to write new `torch.nn.Modules`. We may have to implement new functions usable with autograd, so that `Modules` remain defined through their forward pass alone.
This is achieved by writing sub-classes of `torch.autograd.Function`, which have to implement two static methods:

- `forward(...)` takes as argument a context to store information needed for the backward pass, and the quantities it should process, which are `Tensors` for the differentiable ones, but can also be any other types. It should return one or several `Tensors`. 
This is achieved by writing sub-classes of `torch.autograd.Function`, which have to implement two static methods:

- **forward(...)** takes as argument a context to store information needed for the backward pass, and the quantities it should process, which are Tensors for the differentiable ones, but can also be any other types. It should return one or several Tensors.

- **backward(...)** takes as argument the context and as many Tensors as `forward` returns Tensors, and it should return as many values as `forward` takes argument, Tensors for the tensors and `None` for the others.
This is achieved by writing sub-classes of `torch.autograd.Function`, which have to implement two static methods:

- **forward(...)** takes as argument a context to store information needed for the backward pass, and the quantities it should process, which are `Tensors` for the differentiable ones, but can also be any other types. It should return one or several `Tensors`.

- **backward(...)** takes as argument the context and as many `Tensors` as `forward` returns `Tensors`, and it should return as many values as `forward` takes argument, `Tensors` for the tensors and `None` for the others.

Evaluating such a `Function` is done through its `apply(...)` method, which takes as many arguments as `forward(...`, context excluded.
If you create a new `Function` named `Dummy`, when `Dummy.apply(...)` is called, autograd first adds a new node of type `DummyBackward` in its graph, and then calls `Dummy.forward(...)`. 
If you create a new `Function` named `Dummy`, when `Dummy.apply(...)` is called, autograd first adds a new node of type `DummyBackward` in its graph, and then calls `Dummy.forward(...).`

To compute the gradient, autograd evaluates the graph and calls `Dummy.backward(...)` when it reaches the corresponding node, with the same context as the one given to `Dummy.forward(...).`
If you create a new `Function` named `Dummy`, when `Dummy.apply(...)` is called, autograd first adds a new node of type `DummyBackward` in its graph, and then calls `Dummy.forward(...)`. To compute the gradient, autograd evaluates the graph and calls `Dummy.backward(...)` when it reaches the corresponding node, with the same context as the one given to `Dummy.forward(...)`. This machinery is hidden to you and this level of details should not be required for normal operations.
Consider a function to set to zero the first $n$ components of a tensor.

class KillHead(Function):
    @staticmethod
    def forward(ctx, input, n):
        ctx.n = n
        result = input.clone()
        result[:, 0:ctx.n] = 0
        return result

    @staticmethod
    def backward(ctx, grad_output):
        result = grad_output.clone()
        result[:, 0:ctx.n] = 0
        return result, None

killhead = KillHead.apply
It can be used for instance

```python
y = torch.empty(3, 8).normal_()
x = torch.empty(y.size()).normal_().requires_grad_()

criterion = nn.MSELoss()
optimizer = torch.optim.SGD([x], lr = 1.0)

for k in range(5):
    r = killhead(x, 2)
    loss = criterion(r, y)
    print(k, loss.item())
    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```
It can be used for instance

```python
y = torch.empty(3, 8).normal_()
x = torch.empty(y.size()).normal_().requires_grad_()

criterion = nn.MSELoss()
optimizer = torch.optim.SGD([x], lr = 1.0)

for k in range(5):
    r = killhead(x, 2)
    loss = criterion(r, y)
    print(k, loss.item())

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

prints

```
0 1.5175858736038208
1 1.310139536857605
2 1.1358269453048706
3 0.9893561005592346
4 0.8662799000740051
```
The `torch.autograd.gradcheck(...)` function checks numerically that the backward function is correct, *i.e.*

\[
\forall i, j, \quad \left| \frac{f_i(x_1, \ldots, x_j + \epsilon, \ldots, x_D) - f_i(x_1, \ldots, x_j - \epsilon, \ldots, x_D)}{2\epsilon} - (J_f(x))_{i,j} \right| \leq \alpha
\]
The `torch.autograd.gradcheck(...)` function checks numerically that the backward function is correct, \textit{i.e.}

\[ \forall i, j, \frac{f_i(x_1, \ldots, x_j + \epsilon, \ldots, x_D) - f_i(x_1, \ldots, x_j - \epsilon, \ldots, x_D)}{2\epsilon} - (J_f(x))_{i,j} \leq \alpha \]

\[ x = \text{torch.empty}(10, 20, \text{dtype} = \text{torch.float64}).\text{uniform}_(-1, 1).\text{requires_grad}() \]

\[ \text{input} = (x, 4) \]

if gradcheck(killhead, input, eps = 1e-6, atol = 1e-4):
  print('All good captain.')
else:
  print('Ouch')

It is advisable to use `torch.float64` s for such a check.
The `torch.autograd.gradcheck(...)` function checks numerically that the backward function is correct, i.e.

\[ \forall i, j, \quad \left| \frac{f_i(x_1, \ldots, x_j + \epsilon, \ldots, x_D) - f_i(x_1, \ldots, x_j - \epsilon, \ldots, x_D)}{2\epsilon} - (J_f(x))_{i,j} \right| \leq \alpha \]

\[
x = torch.empty(10, 20, dtype = torch.float64).uniform_(-1, 1).requires_grad_()
\]

\[
\text{input} = (x, 4)
\]

\[
\text{if gradcheck(killhead, input, eps = 1e-6, atol = 1e-4):}
\quad \text{print('All good captain.'})
\]

\[
\text{else:}
\quad \text{print('Ouch')}
\]

⚠️ It is advisable to use `torch.float64` for such a check.
Consider a function that takes two similar sized Tensors and apply component-wise
\[(u, v) \mapsto |uv|\].
Consider a function that takes two similar sized Tensors and apply component-wise

$$(u, v) \mapsto |uv|.$$ 

The backward has to compute two tensors, and the forward must keep track of the input to compute the derivatives in the backward.
Consider a function that takes two similar sized Tensors and apply component-wise
\[(u, v) \mapsto |uv|.
\]
The backward has to compute two tensors, and the forward must keep track of the input to compute the derivatives in the backward.

class Something(Function):
    @staticmethod
    def forward(ctx, input1, input2):
        ctx.save_for_backward(input1, input2)
        return (input1 * input2).abs()

    @staticmethod
    def backward(ctx, grad_output):
        input1, input2 = ctx.saved_tensors
        return grad_output * input1.sign() * input2.abs(),
            grad_output * input1.abs() * input2.sign()

something = Something.apply
The end