

Deep learning

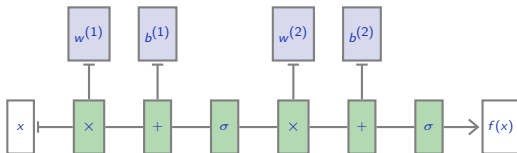
4.1. DAG networks

François Fleuret

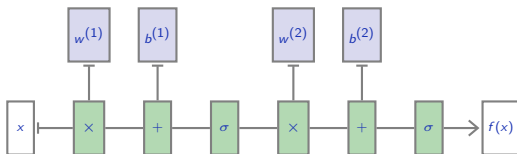
<https://fleuret.org/dlc/>

Dec 20, 2020

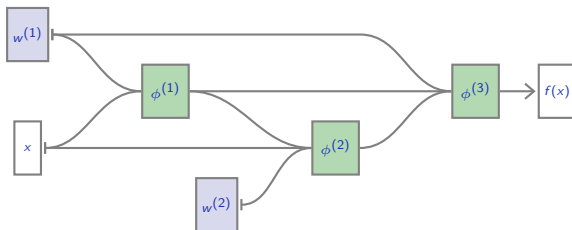
We can generalize an MLP



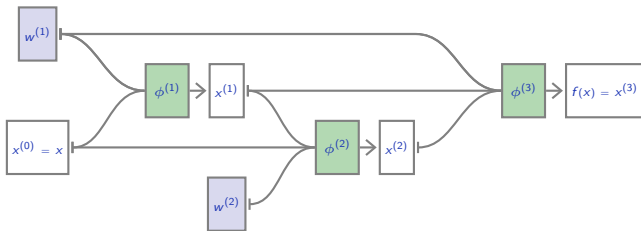
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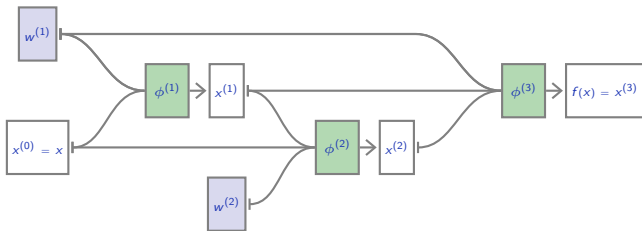
to an arbitrary “Directed Acyclic Graph” (DAG) of operators



Forward pass

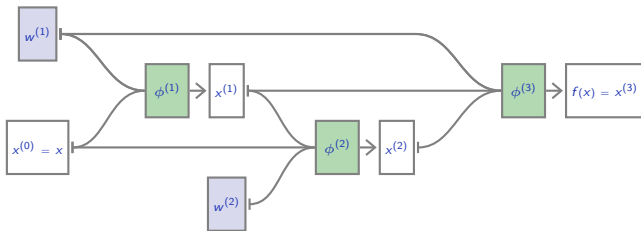


Forward pass



$$x^{(0)} = x$$

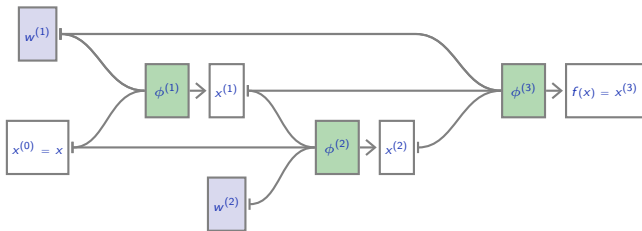
Forward pass



$$x^{(0)} = x$$

$$x^{(1)} = \phi^{(1)}(x^{(0)}; w^{(1)})$$

Forward pass

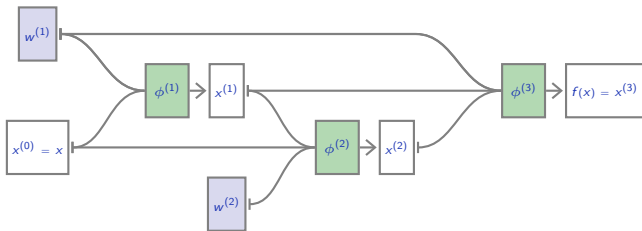


$$x^{(0)} = x$$

$$x^{(1)} = \phi^{(1)}(x^{(0)}; w^{(1)})$$

$$x^{(2)} = \phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)})$$

Forward pass



$$\begin{aligned}x^{(0)} &= x \\x^{(1)} &= \phi^{(1)}(x^{(0)}; w^{(1)}) \\x^{(2)} &= \phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)}) \\f(x) = x^{(3)} &= \phi^{(3)}(x^{(1)}, x^{(2)}; w^{(1)})\end{aligned}$$

If $(a_1, \dots, a_Q) = \phi(b_1, \dots, b_R)$, we use the notation

$$\begin{bmatrix} \frac{\partial a}{\partial b} \end{bmatrix} = J_\phi = \begin{pmatrix} \frac{\partial a_1}{\partial b_1} & \cdots & \frac{\partial a_1}{\partial b_R} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_Q}{\partial b_1} & \cdots & \frac{\partial a_Q}{\partial b_R} \end{pmatrix}.$$

It does not specify at which point this is computed, but it will always be for the forward-pass activations.

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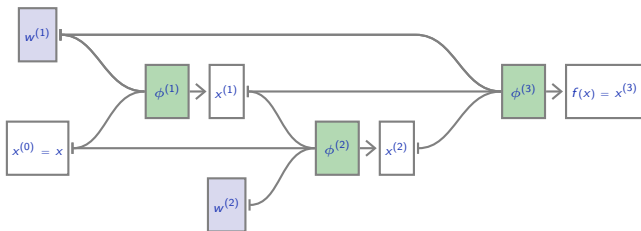
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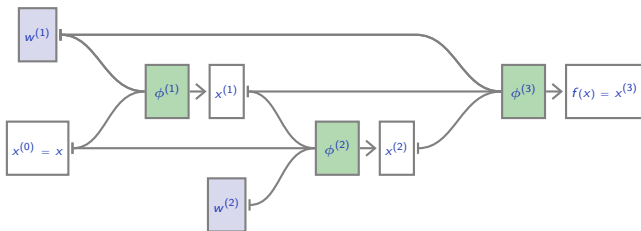
Also, if $(a_1, \dots, a_Q) = \phi(b_1, \dots, b_R, c_1, \dots, c_S)$, we use

$$\begin{bmatrix} \frac{\partial a}{\partial c} \end{bmatrix} = J_{\phi|c} = \begin{pmatrix} \frac{\partial a_1}{\partial c_1} & \cdots & \frac{\partial a_1}{\partial c_S} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_Q}{\partial c_1} & \cdots & \frac{\partial a_Q}{\partial c_S} \end{pmatrix}.$$

Backward pass, derivatives w.r.t activations

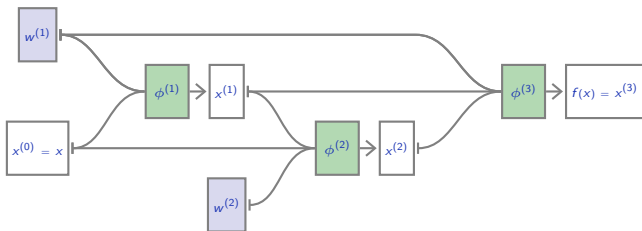


Backward pass, derivatives w.r.t activations



$$\left[\frac{\partial \ell}{\partial x^{(2)}} \right] = \left[\frac{\partial x^{(3)}}{\partial x^{(2)}} \right] \left[\frac{\partial \ell}{\partial x^{(3)}} \right] = J_{\phi^{(3)}|x^{(2)}} \left[\frac{\partial \ell}{\partial x^{(3)}} \right]$$

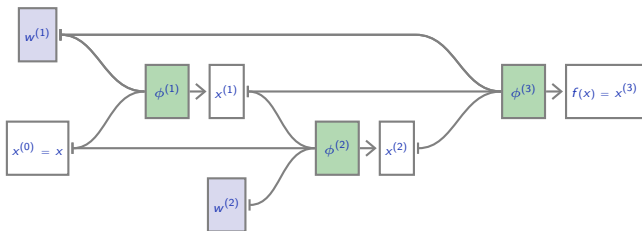
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$$\left[\frac{\partial \ell}{\partial x^{(1)}} \right] = \left[\frac{\partial x^{(2)}}{\partial x^{(1)}} \right] \left[\frac{\partial \ell}{\partial x^{(2)}} \right] + \left[\frac{\partial x^{(3)}}{\partial x^{(1)}} \right] \left[\frac{\partial \ell}{\partial x^{(3)}} \right] = J_{\phi^{(2)}|x^{(1)}} \left[\frac{\partial \ell}{\partial x^{(2)}} \right] + J_{\phi^{(3)}|x^{(1)}} \left[\frac{\partial \ell}{\partial x^{(3)}} \right]$$

Backward pass, derivatives w.r.t activations

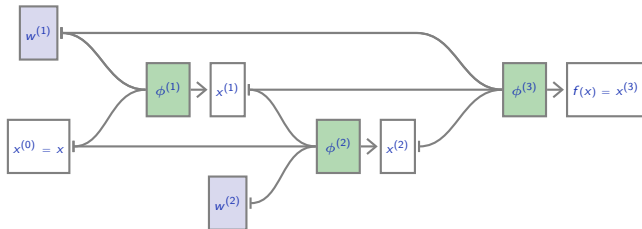


$$\left[\frac{\partial \ell}{\partial \mathbf{x}^{(2)}} \right] = \left[\frac{\partial \mathbf{x}^{(3)}}{\partial \mathbf{x}^{(2)}} \right] \left[\frac{\partial \ell}{\partial \mathbf{x}^{(3)}} \right] = J_{\phi^{(3)}|\mathbf{x}^{(2)}} \left[\frac{\partial \ell}{\partial \mathbf{x}^{(3)}} \right]$$

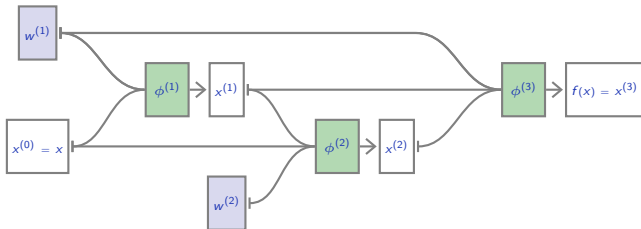
$$\left[\frac{\partial \ell}{\partial \mathbf{x}^{(1)}} \right] = \left[\frac{\partial \mathbf{x}^{(2)}}{\partial \mathbf{x}^{(1)}} \right] \left[\frac{\partial \ell}{\partial \mathbf{x}^{(2)}} \right] + \left[\frac{\partial \mathbf{x}^{(3)}}{\partial \mathbf{x}^{(1)}} \right] \left[\frac{\partial \ell}{\partial \mathbf{x}^{(3)}} \right] = J_{\phi^{(2)}|\mathbf{x}^{(1)}} \left[\frac{\partial \ell}{\partial \mathbf{x}^{(2)}} \right] + J_{\phi^{(3)}|\mathbf{x}^{(1)}} \left[\frac{\partial \ell}{\partial \mathbf{x}^{(3)}} \right]$$

$$\left[\frac{\partial \ell}{\partial \mathbf{x}^{(0)}} \right] = \left[\frac{\partial \mathbf{x}^{(1)}}{\partial \mathbf{x}^{(0)}} \right] \left[\frac{\partial \ell}{\partial \mathbf{x}^{(1)}} \right] + \left[\frac{\partial \mathbf{x}^{(2)}}{\partial \mathbf{x}^{(0)}} \right] \left[\frac{\partial \ell}{\partial \mathbf{x}^{(2)}} \right] = J_{\phi^{(1)}|\mathbf{x}^{(0)}} \left[\frac{\partial \ell}{\partial \mathbf{x}^{(1)}} \right] + J_{\phi^{(2)}|\mathbf{x}^{(0)}} \left[\frac{\partial \ell}{\partial \mathbf{x}^{(2)}} \right]$$

Backward pass, derivatives w.r.t parameters

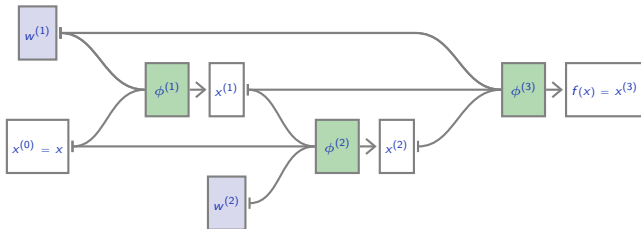


Backward pass, derivatives w.r.t parameters



$$\left[\frac{\partial \ell}{\partial w^{(1)}} \right] = \left[\frac{\partial x^{(1)}}{\partial w^{(1)}} \right] \left[\frac{\partial \ell}{\partial x^{(1)}} \right] + \left[\frac{\partial x^{(3)}}{\partial w^{(1)}} \right] \left[\frac{\partial \ell}{\partial x^{(3)}} \right] = J_{\phi^{(1)}|w^{(1)}} \left[\frac{\partial \ell}{\partial x^{(1)}} \right] + J_{\phi^{(3)}|w^{(1)}} \left[\frac{\partial \ell}{\partial x^{(3)}} \right]$$

Backward pass, derivatives w.r.t parameters



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$$\left[\frac{\partial \ell}{\partial w^{(2)}} \right] = \left[\frac{\partial x^{(2)}}{\partial w^{(2)}} \right] \left[\frac{\partial \ell}{\partial x^{(2)}} \right] = J_{\phi^{(2)}|w^{(2)}} \left[\frac{\partial \ell}{\partial x^{(2)}} \right]$$

So if we have a library of “tensor operators”, and implementations of

$$\begin{aligned} & (x_1, \dots, x_d, w) \mapsto \phi(x_1, \dots, x_d; w) \\ \forall c, & (x_1, \dots, x_d, w) \mapsto J_{\phi|x_c}(x_1, \dots, x_d; w) \\ & (x_1, \dots, x_d, w) \mapsto J_{\phi|w}(x_1, \dots, x_d; w), \end{aligned}$$

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$$\begin{aligned}(x_1, \dots, x_d, w) &\mapsto \phi(x_1, \dots, x_d; w) \\ \forall c, (x_1, \dots, x_d, w) &\mapsto J_{\phi|_{x_c}}(x_1, \dots, x_d; w) \\ (x_1, \dots, x_d, w) &\mapsto J_{\phi|_w}(x_1, \dots, x_d; w),\end{aligned}$$

we can build an arbitrary directed acyclic graph with these operators at the nodes, compute the response of the resulting mapping, and compute its gradient with back-prop.

Writing from scratch a large neural network is complex and error-prone.

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Multiple frameworks provide libraries of tensor operators and mechanisms to combine them into DAGs and automatically differentiate them.

	Language(s)	License	Main backer
PyTorch	Python	BSD	Facebook
Caffe2	C++, Python	Apache	Facebook
TensorFlow	Python, C++	Apache	Google
JAX	Python	Apache	Google
MXNet	Python, C++, R, Scala	Apache	Amazon
CNTK	Python, C++	MIT	Microsoft
Torch	Lua	BSD	Facebook
Theano	Python	BSD	U. of Montreal
Caffe	C++	BSD 2 clauses	U. of CA, Berkeley

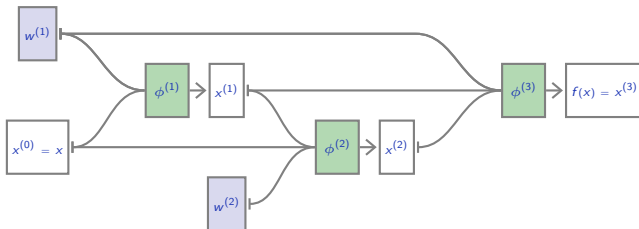
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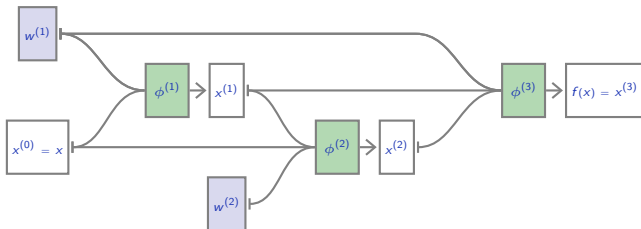
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MXNet	Python, C++, R, Scala	Apache	Amazon
CNTK	Python, C++	MIT	Microsoft
Torch	Lua	BSD	Facebook
Theano	Python	BSD	U. of Montreal
Caffe	C++	BSD 2 clauses	U. of CA, Berkeley

One approach is to define the nodes and edges of such a DAG statically (Torch, TensorFlow, Caffe, Theano, etc.)

In TensorFlow, to run a forward/backward pass on



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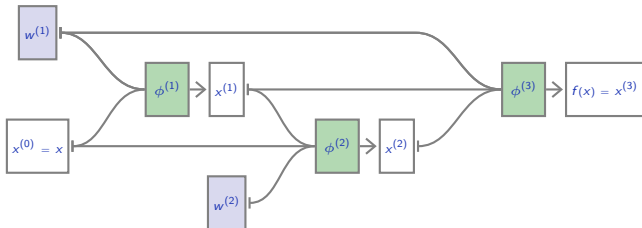


$$\phi^{(1)}(x^{(0)}; w^{(1)}) = w^{(1)}x^{(0)}$$

$$\phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)}) = x^{(0)} + w^{(2)}x^{(1)}$$

$$\phi^{(3)}(x^{(1)}, x^{(2)}; w^{(1)}) = w^{(1)}(x^{(1)} + x^{(2)})$$

In TensorFlow, to run a forward/backward pass on



$$\begin{aligned}\phi^{(1)}(x^{(0)}; w^{(1)}) &= w^{(1)}x^{(0)} \\ \phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)}) &= x^{(0)} + w^{(2)}x^{(1)} \\ \phi^{(3)}(x^{(1)}, x^{(2)}; w^{(1)}) &= w^{(1)}(x^{(1)} + x^{(2)})\end{aligned}$$

```
w1 = tf.Variable(tf.random_normal([5, 5]))
w2 = tf.Variable(tf.random_normal([5, 5]))
x = tf.Variable(tf.random_normal([5, 1]))
x0 = x
x1 = tf.matmul(w1, x0)
x2 = x0 + tf.matmul(w2, x1)
x3 = tf.matmul(w1, x1 + x2)
q = tf.norm(x3)
```

```
gw1, gw2 = tf.gradients(q, [w1, w2])
```

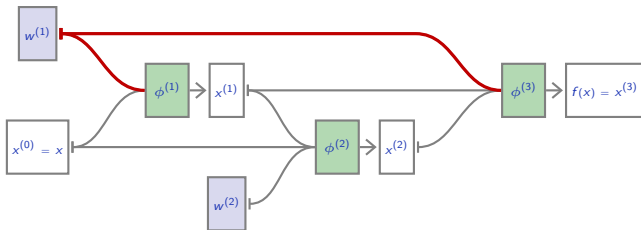
```
with tf.Session() as sess:
    sess.run(tf.global_variables_initializer())
    _gw1, _gw2 = sess.run([gw1, gw2])
```

Weight sharing

In our generalized DAG formulation, we have in particular implicitly allowed the same parameters to modulate different parts of the processing.

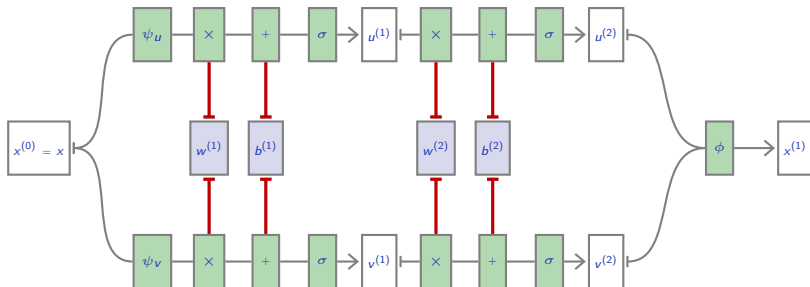
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For instance $w^{(1)}$ in our example parametrizes both $\phi^{(1)}$ and $\phi^{(3)}$.

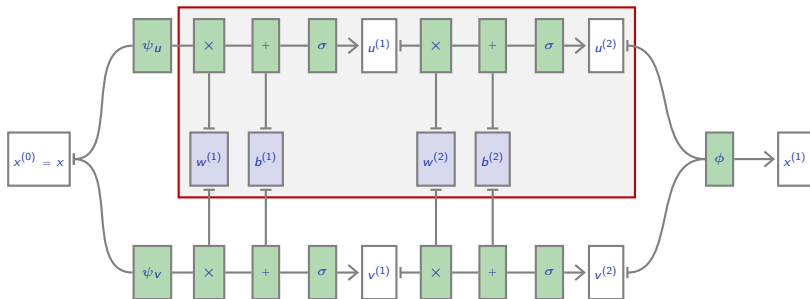


This is called **weight sharing**.

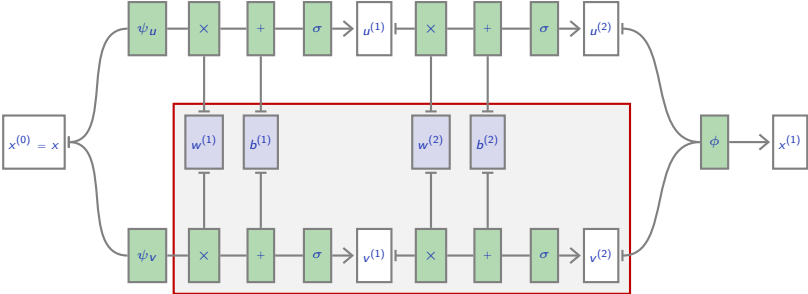
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The end