Deep learning

3.3. Linear separability and feature design

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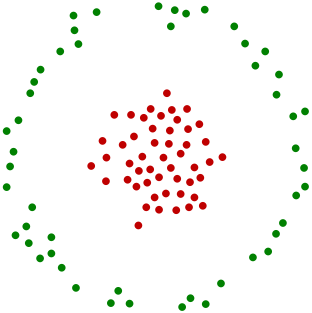

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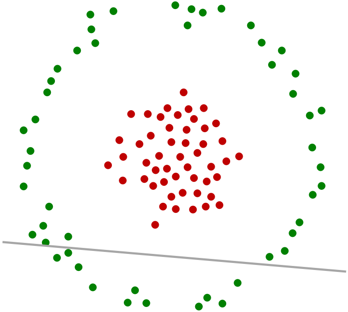

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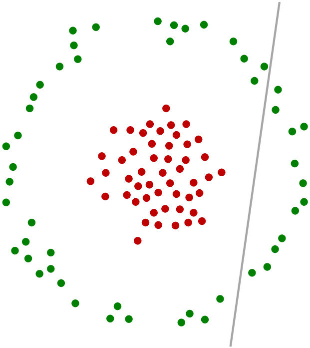

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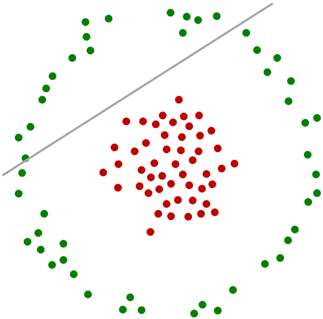

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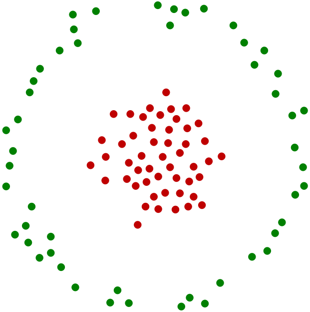
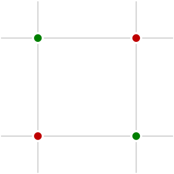
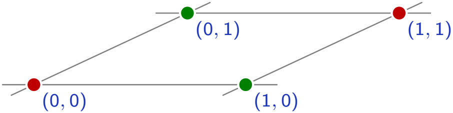




“xor”

The xor example can be solved by pre-processing the data to make the two populations linearly separable.



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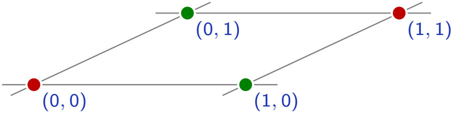
$$\Phi : (x_u, x_v) \mapsto (x_u, x_v, x_u x_v).$$



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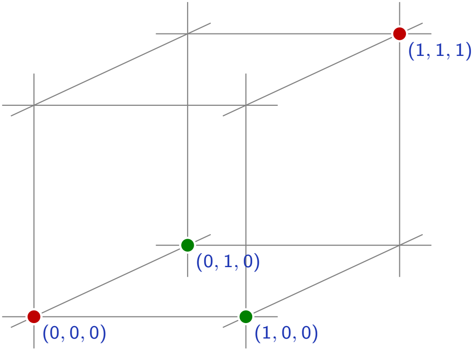

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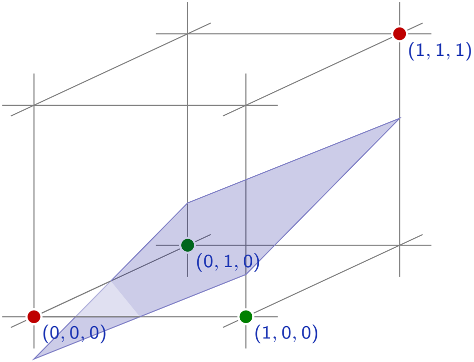

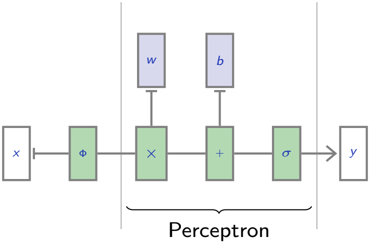
x
Φ
w
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Perceptron

This is similar to the polynomial regression. If we have

$$\Phi : x \mapsto (1, x, x^2, \dots, x^D)$$

and

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By increasing D, we can approximate any continuous real function on a compact space (Stone-Weierstrass theorem).

It means that we can make the capacity as high as we want.

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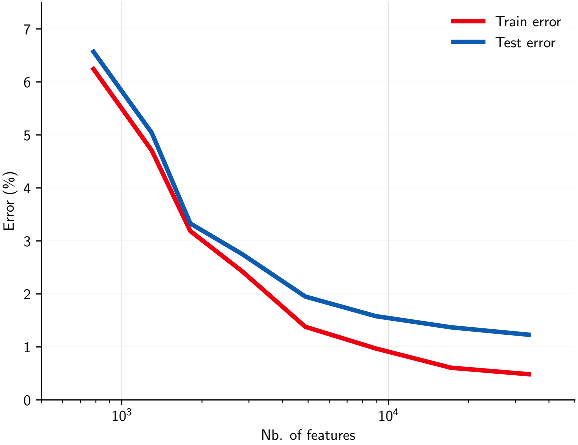

Remember the bias-variance tradeoff:

$$\mathbb{E}((Y-y)^2) = \underbrace{(\mathbb{E}(Y)-y)^2}_{\text{Bias}} + \underbrace{\mathbb{V}(Y)}_{\text{Variance}}.$$

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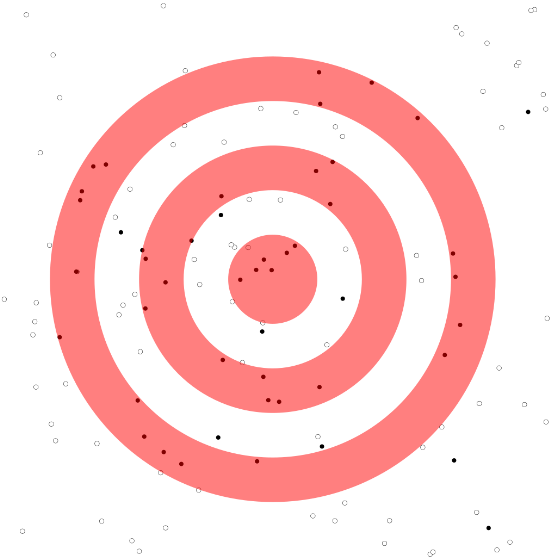
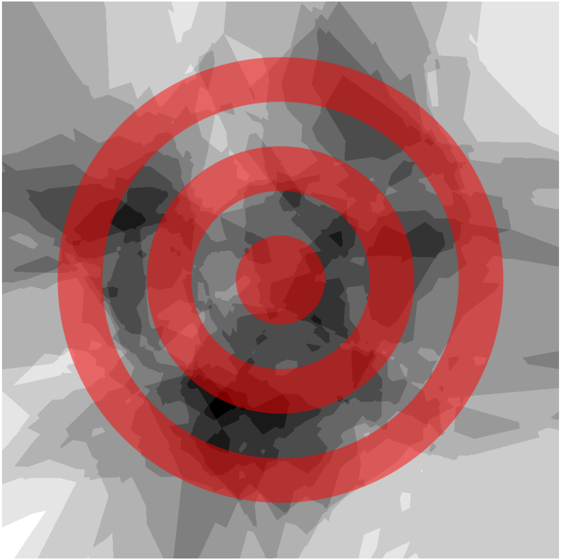
Beside increasing capacity to reduce the bias, "feature design" may also be a way of reducing capacity without hurting the bias, or with improving it.

In particular, good features should be invariant to perturbations of the signal known to keep the value to predict unchanged.

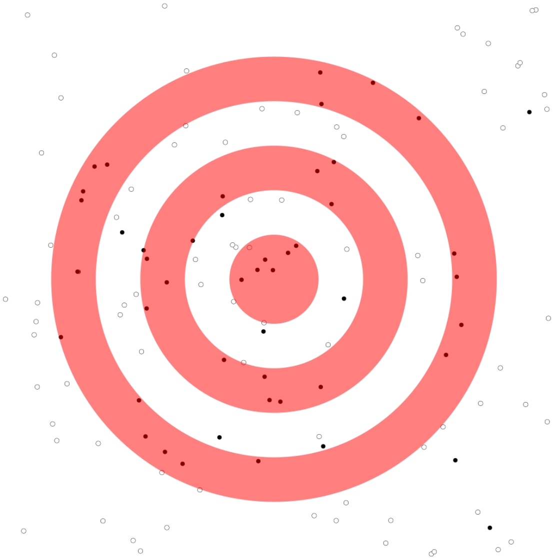


Training points



Votes (K=11)

Prediction (K=11)



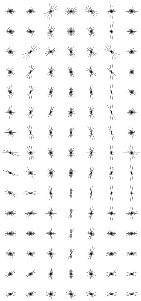
Training points

Votes, radial feature (K=11)

Prediction, radial feature (K=11)

A classical example is the "Histogram of Oriented Gradient" descriptors (HOG), initially designed for person detection.

Roughly: divide the image in 8×8 blocks, compute in each the distribution of edge orientations over 9 bins.



Dalal and Triggs (2005) combined them with a SVM, and Dollár et al. (2009) extended them with other modalities into the "channel features".

Many methods (perceptron, SVM, k-means, PCA, etc.) only require to compute $\kappa(x, x') = \Phi(x) \cdot \Phi(x')$ for any (x, x').

So one needs to specify κ alone, and may keep Φ undefined.

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This is the **kernel trick**, which we will not talk about in this course.

Training a model composed of manually engineered features and a parametric model such as logistic regression is now referred to as **“shallow learning”**.

The signal goes through a single processing trained from data.

The end

References

N. Dalal and B. Triggs. **Histograms of oriented gradients for human detection**. In Conference on Computer Vision and Pattern Recognition (CVPR), pages 886–893, 2005.

P. Dollár, Z. Tu, P. Perona, and S. Belongie. **Integral channel features**. In British Machine Vision Conference, pages 91.1–91.11, 2009.