Deep learning

1.5. High dimension tensors

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https://fleuret.org/dlc/

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A tensor can be of several types:

- `torch.float16`, `torch.float32`, `torch.float64`,
- `torch.uint8`,
- `torch.int8`, `torch.int16`, `torch.int32`, `torch.int64`

and can be located either in the CPU’s or in a GPU’s memory.

Operations with tensors stored in a certain device’s memory are done by that device. We will come back to that later.
```python
>>> x = torch.zeros(1, 3)
>>> x.dtype, x.device
(torch.float32, device(type='cpu'))
>>> x = x.long()
>>> x.dtype, x.device
(torch.int64, device(type='cpu'))
>>> x = x.to('cuda')
>>> x.dtype, x.device
(torch.int64, device(type='cuda', index=0))
```
2d tensor (e.g. grayscale image)
3d tensor (e.g. rgb image)
4d tensor (e.g. sequence of rgb images)
Here are a few examples from the immense library of tensor operations:

**Creation**

- `torch.empty(*size, ...)`
- `torch.zeros(*size, ...)`
- `torch.full(size, value, ...)`
- `torch.tensor(sequence, ...)`
- `torch.eye(n, ...)`
- `torch.from_numpy(ndarray)`

**Indexing, Slicing, Joining, Mutating**

- `torch.Tensor.view(*size)`
- `torch.cat(inputs, dimension=0)`
- `torch.chunk(tensor, nb_chunks, dim=0)[source]`
- `torch.split(tensor, split_size, dim=0)[source]`
- `torch.index_select(input, dim, index, out=None)`
- `torch.t(input, out=None)`
- `torch.transpose(input, dim0, dim1, out=None)`

**Filling**

- `Tensor.fill_(value)`
- `torch.bernoulli_(_proba)`
- `torch.normal_([mu, [std]])`
**Pointwise math**

- `torch.abs(input, out=None)`
- `torch.add()`
- `torch.cos(input, out=None)`
- `torch.sigmoid(input, out=None)`

**Math reduction**

- `torch.dist(input, other, p=2, out=None)`
- `torch.mean()`
- `torch.norm()`
- `torch.std()`
- `torch.sum()`

**BLAS and LAPACK Operations**

- `torch.eig(a, eigenvectors=False, out=None)`
- `torch.lstsq(B, A, out=None)`
- `torch.inverse(input, out=None)`
- `torch.mm(mat1, mat2, out=None)`
- `torch.mv(mat, vec, out=None)`
\[ x = \text{torch.tensor}(\begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & 6 \end{bmatrix}) \]

\[ x.t() \]
\[ x = \text{torch.tensor}([[1, 3, 0], [2, 4, 6]]) \]

\[ x.\text{view}(-1) \]
\[
x = \text{torch.tensor([[1, 3, 0], [2, 4, 6]])}
\]

\[
x.\text{view}(3, -1)
\]
```python
x = torch.tensor([[1, 3, 0],
                  [2, 4, 6]])

x[:, 1:3]
```

```python
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```
\[ x = \text{torch.tensor}([ [ 1, 3, 0 ], [ 2, 4, 6 ] ]) \]

\[ x.\text{view}(1, 2, 3).\text{expand}(3, 2, 3) \]
\[
x = \text{torch.tensor([[[1, 2, 1],
                        [2, 1, 2]],[
                        [3, 0, 3],
                        [0, 3, 0]]])
\]

\[
x[0:1, :, :]
\]
```python
x = torch.tensor(
    [[[ 1, 2, 1 ],
      [ 2, 1, 2 ]],
     [[ 3, 0, 3 ],
      [ 0, 3, 0 ]]])

x[:, :, 0:2]
```

```
x[:, :, 0:2]
```
x = torch.tensor(
    [[
      [1, 2, 1],
      [2, 1, 2],
    ],
    [[3, 0, 3],
      [0, 3, 0]]
))
\[
x = \text{torch.tensor}([[ [ 1, 2, 1 ],
                       [ 2, 1, 2 ] ],
                       [ [ 3, 0, 3 ],
                       [ 0, 3, 0 ] ]]])
\]
\[
x.\text{transpose}(0, 2)
\]
```python
x = torch.tensor([[[1, 2, 1],
                  [2, 1, 2]],
                  [[[3, 0, 3],
                  [0, 3, 0]]])
x.transpose(1, 2)
```

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x = torch.tensor([[[1, 2, 1],
                  [2, 1, 2]],
                  [[[3, 0, 3],
                  [0, 3, 0]]])
x.transpose(1, 2)
```
PyTorch offers simple interfaces to standard image data-bases.

```python
import torch, torchvision
cifar = torchvision.datasets.CIFAR10('./cifar10/', train = True, download = True)
x = torch.from_numpy(cifar.data).permute(0, 3, 1, 2).float() / 255
print(x.dtype, x.size(), x.min().item(), x.max().item())
```

Printing...

```
Files already downloaded and verified
torch.float32 torch.Size([50000, 3, 32, 32]) 0.0 1.0
```

[...]

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prints

Files already downloaded and verified
torch.float32 torch.Size([50000, 3, 32, 32]) 0.0 1.0
```
# Narrows to the first images, converts to float
x = x[:48]

# Saves these samples as a single image
torchvision.utils.save_image(x, 'cifar-4x12.png',
    nrow = 12, pad_value = 1.0)
# Switches the row and column indexes
x.transpose_(2, 3)
torchvision.utils.save_image(x, 'cifar-4x12-rotated.png',
    nrow = 12, pad_value = 1.0)
# Kills the green and blue channels

```python
x[:, 1:3].fill_(0)
torchvision.utils.save_image(x, 'cifar-4x12-rotated-and-red.png',
                           nrow = 12, pad_value = 1.0)
```
Broadcasting and Einstein summations
**Broadcasting** automagically expands dimensions by replicating coefficients, when it is necessary to perform operations that are “intuitively reasonable”.

```python
>>> x = torch.empty(100, 4).normal_(2)
>>> x.mean(0)
tensor([2.0476, 2.0133, 1.9109, 1.8588])
>>> x -= x.mean(0)  # This should not work, but it does!
>>> x.mean(0)
tensor([-4.0531e-08, -4.4703e-07, -1.3471e-07, 3.5763e-09])
```
**Broadcasting** automagically expands dimensions by replicating coefficients, when it is necessary to perform operations that are “intuitively reasonable”.

For instance:

```python
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>>> x.mean(0)
tensor([-4.0531e-08, -4.4703e-07, -1.3471e-07, 3.5763e-09])
```
Precisely, broadcasting proceeds as follows:

1. If one of the tensors has fewer dimensions than the other, it is reshaped by adding as many dimensions of size 1 as necessary in the front; then

2. for every dimension mismatch, if one of the two tensors is of size one, it is expanded along this axis by replicating coefficients.

If there is a tensor size mismatch for one of the dimension and neither of them is one, the operation fails.
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]])
C = A + B
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]])
C = A + B
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]]).
C = A + B

Broadcasted

\[
\begin{align*}
A & = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \\
B & = \begin{bmatrix} 5 & -5 & 5 & -5 & 5 \\ 5 & -5 & 5 & -5 & 5 \\ 5 & -5 & 5 & -5 & 5 \end{bmatrix}
\end{align*}
\]

replicate

\[
\begin{align*}
A & \vdash \overset{4}{4} \overset{3}{3} \overset{2}{2} \overset{1}{1} \\
B & \vdash \begin{bmatrix} 6 & -4 & 6 & -4 & 6 \\ 7 & -3 & 7 & -3 & 7 \\ 8 & -2 & 8 & -2 & 8 \\ 9 & -1 & 9 & -1 & 9 \end{bmatrix}
\end{align*}
\]

replicate

\[C = A + B\]
A powerful generic tool for complex tensorial operations is the Einstein summation convention. It provides a concise way of describing dimension re-ordering and summing of component-wise products along some of them.

`torch.einsum` takes as argument a string describing the operation, the tensors to operate on, and returns a tensor.

The operation string is a comma-separated list of indexing, followed by the indexing for the result.

**Summations are executed on all indexes not appearing in the result indexing.**
For instance, we can formulate that way the standard matrix product:

\[
\mathbb{R}^{A \times B} \times \mathbb{R}^{B \times C} \rightarrow \mathbb{R}^{A \times C}
\]

\[
\forall i, k, m_{i,k} = \sum_j p_{i,j} q_{j,k}
\]

\[
m = \text{torch.einsum('ij,jk->ik', p, q)}
\]

The summation is done along \( j \) since it does not appear after the \( \rightarrow \).
For instance, we can formulate that way the standard matrix product:

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\[m = \text{torch.einsum}('ij,jk->ik', p, q)\]

The summation is done along \( j \) since it does not appear after the \( \rightarrow \).

```python
>>> p = torch.rand(2, 5)
>>> q = torch.rand(5, 4)
>>> torch.einsum('ij,jk->ik', p, q)
tensor([[2.0833, 1.1046, 1.5220, 0.4405],
        [2.1338, 1.2601, 1.4226, 0.8641]])
```

```python
>>> p@q
tensor([[2.0833, 1.1046, 1.5220, 0.4405],
        [2.1338, 1.2601, 1.4226, 0.8641]])
```
Matrix-vector product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^B \rightarrow \mathbb{R}^A \]

\[ \forall i, k, w_i = \sum_j m_{i,j} v_j \]

\[ w = \text{torch.einsum}(\text{\textquoteleft}ij,j\rightarrow\text{\textquoteleft}i\text{\textclosequote}, m, v) \]
Matrix-vector product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^B \rightarrow \mathbb{R}^A \]

\[ \forall i, k, \, w_i = \sum_j m_{i,j} v_j \]

\[ w = \text{torch.einsum('ij,j->i', m, v) \]
Matrix-vector product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^B \rightarrow \mathbb{R}^A \]

\[ \forall i, k, \ w_i = \sum_j m_{i,j} v_j \]

\[ w = \text{torch.einsum}(\text{'ij,j->i'}, m, v) \]

Hadamard (component-wise) product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^{A \times B} \rightarrow \mathbb{R}^{A \times B} \]

\[ \forall i, j, \ m_{i,j} = p_{i,j} q_{i,j} \]

\[ m = \text{torch.einsum}(\text{'ij,ij->ij'}, p, q) \]

Extracting the diagonal:

\[ \mathbb{R}^{D \times D} \rightarrow \mathbb{R}^D \]

\[ \forall i, k, \ v_i = m_{i,i} \]

\[ v = \text{torch.einsum}(\text{'ii->i'}, m) \]
Batch matrix product:

\[
\mathbb{R}^{N \times A \times B} \times \mathbb{R}^{N \times B \times C} \rightarrow \mathbb{R}^{N \times A \times C}
\]

\[
\forall n, i, k, \quad m_{n, i, k} = \sum_j p_{n, i, j} q_{n, j, k}
\]

\[
m = \text{torch.einsum('nij,njk->nik', } p, q)\]
Batch matrix product:

\[
\mathbb{R}^{N \times A \times B} \times \mathbb{R}^{N \times B \times C} \rightarrow \mathbb{R}^{N \times A \times C}
\]

\[\forall n, i, k, \ m_{n,i,k} = \sum_j p_{n,i,j} q_{n,j,k}\]

\[m = \text{torch.einsum('nij,njk->nik', p, q)}\]

Batch trace:

\[
\mathbb{R}^{N \times D \times D} \rightarrow \mathbb{R}^{N}
\]

\[\forall n, \ t_n = \sum_i m_{n,i,i}\]

\[t = \text{torch.einsum('nii->n', m)}\]
Batch matrix product:

\[
\mathbb{R}^{N \times A \times B} \times \mathbb{R}^{N \times B \times C} \rightarrow \mathbb{R}^{N \times A \times C}
\]

\[
\forall n, i, k, \ m_{n,i,k} = \sum_j p_{n,i,j} q_{n,j,k}
\]

\[
m = \text{torch.einsum}(\text{'nij,njk->nik'}, p, q)
\]

Batch trace:

\[
\mathbb{R}^{N \times D \times D} \rightarrow \mathbb{R}^{N}
\]

\[
\forall n, \ t_n = \sum_i m_{n,i,i}
\]

\[
t = \text{torch.einsum}(\text{'nii->n'}, m)
\]

Tri-linear product along a channel:

\[
\mathbb{R}^{N \times C \times T} \times \mathbb{R}^{N \times C \times T} \times \mathbb{R}^{N \times C \times T} \rightarrow \mathbb{R}^{N \times T}
\]

\[
\forall n, t, \ m_{n,t} = \sum_c p_{n,c,t} q_{n,c,t} r_{n,c,t}
\]

\[
m = \text{torch.einsum}(\text{'nct,nct,nct->nt'}, p, q, r)
\]
The end