Deep learning

1.5. High dimension tensors

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A tensor can be of several types:

- `torch.float16`, `torch.float32`, `torch.float64`,
- `torch.uint8`,
- `torch.int8`, `torch.int16`, `torch.int32`, `torch.int64`

and can be located either in the CPU’s or in a GPU’s memory.

Operations with tensors stored in a certain device’s memory are done by that device. We will come back to that later.
```python
>>> x = torch.zeros(1, 3)
>>> x.dtype, x.device
torch.float32, device(type='cpu'))
>>> x = x.long()
>>> x.dtype, x.device
torch.int64, device(type='cpu'))
>>> x = x.to('cuda')
>>> x.dtype, x.device
torch.int64, device(type='cuda', index=0))
```
2d tensor (e.g. grayscale image)
3d tensor (e.g. rgb image)
4d tensor (e.g. sequence of rgb images)
Here are a few examples from the immense library of tensor operations:

**Creation**

- `torch.empty(*size, ...)`
- `torch.zeros(*size, ...)`
- `torch.full(size, value, ...)`
- `torch.tensor(sequence, ...)`
- `torch.eye(n, ...)`
- `torch.from_numpy(ndarray)`

**Indexing, Slicing, Joining, Mutating**

- `torch.Tensor.view(*size)`
- `torch.cat(inputs, dimension=0)`
- `torch.chunk(tensor, nb_chunks, dim=0)[source]`
- `torch.split(tensor, split_size, dim=0)[source]`
- `torch.index_select(input, dim, index, out=None)`
- `torch.t(input, out=None)`
- `torch.transpose(input, dim0, dim1, out=None)`

**Filling**

- `Tensor.fill_(value)`
- `torch.bernoulli_(proba)`
- `torch.normal_([mu, [std]])`
Pointwise math

- `torch.abs(input, out=None)`
- `torch.add()`
- `torch.cos(input, out=None)`
- `torch.sigmoid(input, out=None)`

Math reduction

- `torch.dist(input, other, p=2, out=None)`
- `torch.mean()`
- `torch.norm()`
- `torch.std()`
- `torch.sum()`

BLAS and LAPACK Operations

- `torch.eig(a, eigenvectors=False, out=None)`
- `torch.lstsq(B, A, out=None)`
- `torch.inverse(input, out=None)`
- `torch.mm(mat1, mat2, out=None)`
- `torch.mv(mat, vec, out=None)`
```python
x = torch.tensor([[1, 3, 0],
                  [2, 4, 6]])

x.t()
```
$x = \text{torch.tensor}([[1, 3, 0], [2, 4, 6]])$

$x\text{.view}(-1)$
x = torch.tensor([ [ 1, 3, 0 ],
                  [ 2, 4, 6 ] ]) 

x.view(3, -1)
\[ x = \text{torch.tensor}([[1, 3, 0], [2, 4, 6]]) \]

\[ x[:, 1:3] \]

\[ x[:, 1:3] \]
```python
x = torch.tensor([ [ 1, 3, 0 ],
                   [ 2, 4, 6 ] ])  
x.view(1, 2, 3).expand(3, 2, 3)
```
\[
x = \text{torch.tensor([[[1, 2, 1], [2, 1, 2]], [[3, 0, 3], [0, 3, 0]]])}
\]

\[
x[0:1, :, :]
\]
\begin{align*}
x &= \text{torch.tensor}([ [ [ 1, 2, 1 ], \ \\ 2, 1, 2 ] ], \\
[ [ 3, 0, 3 ], \\
0, 3, 0 ] ]))
\end{align*}
\[ x = \text{torch.tensor}([\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}]) \]

\[ x.\text{transpose}(0, 1) \]
\[
\begin{align*}
x &= \text{torch.tensor}([\begin{bmatrix}1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 0 & 3 \\ 0 & 3 & 0\end{bmatrix}]) \\
x &\quad . \text{transpose}(0, 2)
\end{align*}
\]
\[
x = \text{torch.tensor([[ [ 1, 2, 1 ],
                        [ 2, 1, 2 ] ],
                        [ [ 3, 0, 3 ],
                        [ 0, 3, 0 ] ] ])
\]

\[
x.\text{transpose}(1, 2)
\]
PyTorch offers simple interfaces to standard image data-bases.

```python
import torch, torchvision
cifar = torchvision.datasets.CIFAR10('./cifar10/', train = True, download = True)
x = torch.from_numpy(cifar.data).permute(0, 3, 1, 2).float() / 255
print(x.dtype, x.size(), x.min().item(), x.max().item())
```

Files already downloaded and verified
torch.float32 torch.Size([50000, 3, 32, 32]) 0.0 1.0
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```

prints

```none
Files already downloaded and verified
torch.float32 torch.Size([50000, 3, 32, 32]) 0.0 1.0
```
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```

prints

```
Files already downloaded and verified
torch.float32 torch.Size([50000, 3, 32, 32]) 0.0 1.0
```
# Narrows to the first images, converts to float
x = x[:48]

# Saves these samples as a single image
torchvision.utils.save_image(x, 'cifar-4x12.png',
    nrow = 12, pad_value = 1.0)
# Switches the row and column indexes
x.transpose_(2, 3)
torchvision.utils.save_image(x, 'cifar-4x12-rotated.png',
    nrow = 12, pad_value = 1.0)
# Kills the green and blue channels
x[:, 1:3].fill_(0)
torchvision.utils.save_image(x, 'cifar-4x12-rotated-and-red.png',
    nrow = 12, pad_value = 1.0)
Broadcasting and Einstein summations
**Broadcasting** automagically expands dimensions by replicating coefficients, when it is necessary to perform operations that are “intuitively reasonable”.

```python
>>> x = torch.empty(100, 4).normal_(2)
>>> x.mean(0)
tensor([2.0476, 2.0133, 1.9109, 1.8588])

>>> x -= x.mean(0)  # This should not work, but it does!
>>> x.mean(0)
tensor([-4.0531e-08, -4.4703e-07, -1.3471e-07, 3.5763e-09])
```
**Broadcasting** automagically expands dimensions by replicating coefficients, when it is necessary to perform operations that are "intuitively reasonable".

For instance:

```python
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>>> x.mean(0)
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>>> x -= x.mean(0)  # This should not work, but it does!
>>> x.mean(0)
tensor([-4.0531e-08, -4.4703e-07, -1.3471e-07, 3.5763e-09])
```
Precisely, broadcasting proceeds as follows:

1. If one of the tensors has fewer dimensions than the other, it is reshaped by adding as many dimensions of size 1 as necessary in the front; then

2. for every dimension mismatch, **if one of the two tensors is of size one**, it is expanded along this axis by replicating coefficients.

If there is a tensor size mismatch for one of the dimension and neither of them is one, the operation fails.
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]])
C = A + B
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]])
C = A + B
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]]
C = A + B

Broadcasted

```
replicate
A

1
2
3
4

B

5  -5  5  -5  5

replicate
C = A + B

6   -4  6   -4  6
7   -3  7   -3  7
8   -2  8   -2  8
9   -1  9   -1  9
```
A powerful generic tool for complex tensorial operations is the Einstein summation convention. It provides a concise way of describing dimension re-ordering and summing of component-wise products along some of them.

`torch.einsum` takes as argument a string describing the operation, the tensors to operate on, and returns a tensor.

The operation string is a comma-separated list of indexing, followed by the indexing for the result.

**Summations are executed on all indexes not appearing in the result indexing.**
For instance, we can formulate that way the standard matrix product:

\[
\mathbb{R}^{A \times B} \times \mathbb{R}^{B \times C} \rightarrow \mathbb{R}^{A \times C}
\]

\[
\forall i, k, \ m_{i,k} = \sum_j p_{i,j} q_{j,k}
\]

\[
m = \text{torch.einsum('ij,jk->ik', p, q)}
\]

The summation is done along \( j \) since it does not appear after the ->.
For instance, we can formulate that way the standard matrix product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^{B \times C} \rightarrow \mathbb{R}^{A \times C} \]

\[ \forall i, k, \ m_{i,k} = \sum_j p_{i,j} q_{j,k} \]

\[ m = \text{torch.einsum('ij,jk->ik', p, q)} \]

The summation is done along \( j \) since it does not appear after the \( \rightarrow \).

```python
>>> p = torch.rand(2, 5)
>>> q = torch.rand(5, 4)
>>> torch.einsum('ij,jk->ik', p, q)
tensor([[2.0833, 1.1046, 1.5220, 0.4405],
        [2.1338, 1.2601, 1.4226, 0.8641]])
>>> p@q
tensor([[2.0833, 1.1046, 1.5220, 0.4405],
        [2.1338, 1.2601, 1.4226, 0.8641]])
```
Matrix-vector product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^{B} \rightarrow \mathbb{R}^{A} \]

\[ \forall i, k, \; w_i = \sum_j m_{i,j} v_j \]

\[ w = \text{torch.einsum('ij,j->i', m, v)} \]
Matrix-vector product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^B \rightarrow \mathbb{R}^A \]

\[ \forall i, k, w_i = \sum_j m_{i,j} v_j \]

\( w = \text{torch.einsum('ij,j->i', m, v) } \)

Hadamard (component-wise) product:

\[ \mathbb{R}^{A \times B} \times \mathbb{R}^{A \times B} \rightarrow \mathbb{R}^{A \times B} \]

\[ \forall i, j, m_{i,j} = p_{i,j} q_{i,j} \]

\( m = \text{torch.einsum('ij,ij->ij', p, q) } \)
Matrix-vector product:

\[
\mathbb{R}^{A \times B} \times \mathbb{R}^B \rightarrow \mathbb{R}^A
\]

\[
\forall i, k, w_i = \sum_j m_{i,j} v_j
\]

\[w = \text{torch.einsum}'ij, j->i', m, v)\]

Hadamard (component-wise) product:

\[
\mathbb{R}^{A \times B} \times \mathbb{R}^{A \times B} \rightarrow \mathbb{R}^{A \times B}
\]

\[
\forall i, j, m_{i,j} = p_{i,j} q_{i,j}
\]

\[m = \text{torch.einsum}'ij, ij->ij', p, q)\]

Extracting the diagonal:

\[
\mathbb{R}^{D \times D} \rightarrow \mathbb{R}^D
\]

\[
\forall i, k, v_i = m_{i,i}
\]

\[v = \text{torch.einsum}'ii->i', m)\]
Batch matrix product:

\[
\mathbb{R}^{N \times A \times B} \times \mathbb{R}^{N \times B \times C} \rightarrow \mathbb{R}^{N \times A \times C}
\]

\[
\forall n, i, k, \quad m_{n,i,k} = \sum_j p_{n,i,j} q_{n,j,k}
\]

\[
m = \text{torch.einsum}(\text{'nij,njk->nik'}, \, \text{p, q})
\]
Batch matrix product:

\[ \mathbb{R}^{N \times A \times B} \times \mathbb{R}^{N \times B \times C} \rightarrow \mathbb{R}^{N \times A \times C} \]

\[ \forall n, i, k, \ m_{n,i,k} = \sum_j p_{n,i,j} q_{n,j,k} \]

\[ m = \text{torch.einsum}('nij,njk->nik', p, q) \]

Batch trace:

\[ \mathbb{R}^{N \times D \times D} \rightarrow \mathbb{R}^{N} \]

\[ \forall n, \ t_n = \sum_i m_{n,i,i} \]

\[ t = \text{torch.einsum}('nii->n', m) \]
Batch matrix product:

\[
\mathbb{R}^{N \times A \times B} \times \mathbb{R}^{N \times B \times C} \rightarrow \mathbb{R}^{N \times A \times C}
\]

\[
\forall n, i, k, \ m_{n, i, k} = \sum_{j} p_{n, i, j} q_{n, j, k}
\]

\[
m = \text{torch.einsum('nij,njk->nik', p, q)}
\]

Batch trace:

\[
\mathbb{R}^{N \times D \times D} \rightarrow \mathbb{R}^{N}
\]

\[
\forall n, \ t_{n} = \sum_{i} m_{n, i, i}
\]

\[
t = \text{torch.einsum('nii->n', m)}
\]

Tri-linear product along a channel:

\[
\mathbb{R}^{N \times C \times T} \times \mathbb{R}^{N \times C \times T} \times \mathbb{R}^{N \times C \times T} \rightarrow \mathbb{R}^{N \times T}
\]

\[
\forall n, t, \ m_{n, t} = \sum_{c} p_{n, c, t} q_{n, c, t} r_{n, c, t}
\]

\[
m = \text{torch.einsum('nct,nct,nct->nt', p, q, r)}
\]
The end