Constructing deep generative architectures requires layers to increase the signal dimension, the contrary of what we have done so far with feed-forward networks.

Some generative processes optimize the input, and as such rely on back-propagation to expend the signal from a low-dimension representation to the high-dimension signal space.

The same can be done in the forward pass with transposed convolution layers whose forward operation corresponds to a convolution layer’s backward pass.
Consider a 1d convolution with a kernel $\kappa$

$$y_i = (x \odot \kappa)_i$$

$$= \sum_a x_{i+a-1} \kappa_a$$

$$= \sum_u x_u \kappa_{u-i+1}.$$ 

We get

$$\left[ \frac{\partial \ell}{\partial x} \right]_u = \frac{\partial \ell}{\partial y_i} \frac{\partial y_i}{\partial x_u}$$

$$= \sum_i \frac{\partial \ell}{\partial y_i} \kappa_{u-i+1}.$$ 

which looks a lot like a standard convolution layer, except that the kernel coefficients are visited in reverse order.

This is actually the standard convolution operator from signal processing. If $\ast$ denotes this operation, we have

$$(x \ast \kappa)_i = \sum_a x_a \kappa_{i-a+1}.$$ 

Coming back to the backward pass of the convolution layer, if

$$y = x \odot \kappa$$

then

$$\left[ \frac{\partial \ell}{\partial x} \right] = \left[ \frac{\partial \ell}{\partial y} \right] \ast \kappa.$$
In the deep-learning field, since it corresponds to transposing the weight matrix of the equivalent fully-connected layer, it is called a **transposed convolution**.

\[
\begin{pmatrix}
\kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 & 0 \\
0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 \\
0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 \\
0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 \\
0 & 0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3
\end{pmatrix}^T =
\begin{pmatrix}
\kappa_1 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_1 & 0 & 0 & 0 \\
\kappa_3 & \kappa_2 & \kappa_1 & 0 & 0 \\
0 & \kappa_3 & \kappa_2 & \kappa_1 & 0 \\
0 & 0 & \kappa_3 & \kappa_2 & \kappa_1 \\
0 & 0 & 0 & \kappa_3 & \kappa_2 \\
0 & 0 & 0 & 0 & \kappa_3
\end{pmatrix}
\]

A convolution can be seen as a series of inner products, a transposed convolution can be seen as a weighted sum of translated kernels.
$F_{\text{conv}}_{\text{transpose}}$ implements the operation we just described. It takes as input a batch of multi-channel samples, and produces a batch of multi-channel samples.

```python
>>> x = torch.tensor([[[0., 0., 1., 0., 0., 0., 0.]]])
>>> k = torch.tensor([[[1., 2., 3.]]])
>>> F.conv1d(x, k)
tensor([[[ 3., 2., 1., 0., 0.]]])
```

$\ast$

```python
>>> F.conv_transpose1d(x, k)
tensor([[[ 0., 0., 1., 2., 3., 0., 0., 0., 0.]]])
```

$\otimes$
The class `nn.ConvTranspose1d` embeds that operation into a `nn.Module`.

```python
>>> x = torch.tensor([[ 1., 0., 0., 0., -1.]]
>>> m = nn.ConvTranspose1d(1, 1, kernel_size=3)
>>> with torch.autograd.no_grad():
...     m.bias.zero_()
...     m.weight.copy_(torch.tensor([ 1, 2, 1 ]))
... Parameter containing:
tensor([0.], requires_grad=True)
Parameter containing:
tensor([[1., 2., 1.]], requires_grad=True)
>>> y = m(x)
>>> y
tensor([[1., 2., 1., 0., -1., -2., -1.]], grad_fn=<SqueezeBackward1>)
```

Transposed convolutions also have a dilation parameter that behaves as for convolution and expends the kernel size without increasing the number of parameters by making it sparse.

They also have a stride and padding parameters, however, due to the relation between convolutions and transposed convolutions:

⚠️ While for convolutions stride and padding are defined in the input map, for transposed convolutions these parameters are defined in the output map, and the latter modulates a cropping operation.
The composition of a convolution and a transposed convolution of same parameters keep the signal size [roughly] unchanged.

A convolution with a stride greater than one may ignore parts of the signal. Its composition with the corresponding transposed convolution generates a map of the size of the observed area.

For instance, a 1d convolution of kernel size \( w \) and stride \( s \) composed with the transposed convolution of same parameters maintains the signal size \( W \), only if

\[
\exists q \in \mathbb{N}, \ W = w + s \cdot q.
\]
It has been observed that transposed convolutions may create some grid-structure artifacts, since generated pixels are not all covered similarly.

For instance with a $4 \times 4$ kernel and stride 3

An alternative is to use an analytic up-scaling, implemented in the PyTorch functional `F.interpolate`.

```python
>>> x = torch.tensor([[[ 1., 2. ], [ 3., 4. ]]]
>>> F.interpolate(x, scale_factor = 3, mode = 'bilinear')
tensor([[[1.0000, 1.0000, 1.3333, 1.6667, 2.0000, 2.0000],
        [1.0000, 1.0000, 1.3333, 1.6667, 2.0000, 2.0000],
        [1.6667, 1.6667, 2.0000, 2.3333, 2.6667, 2.6667],
        [2.3333, 2.3333, 2.6667, 3.0000, 3.3333, 3.3333],
        [3.0000, 3.0000, 3.3333, 3.6667, 4.0000, 4.0000],
        [3.0000, 3.0000, 3.3333, 3.6667, 4.0000, 4.0000]]])

>>> F.interpolate(x, scale_factor = 3, mode = 'nearest')
tensor([[[1., 1., 1., 2., 2., 2.],
        [1., 1., 1., 2., 2., 2.],
        [1., 1., 1., 2., 2., 2.],
        [3., 3., 3., 4., 4., 4.],
        [3., 3., 3., 4., 4., 4.],
        [3., 3., 3., 4., 4., 4.]]])
```
Such module is usually combined with a convolution to learn local corrections to undesirable artifacts of the up-scaling.

In practice, a transposed convolution such as

```python
tconv = nn.ConvTranspose2d(nic, noc,
    kernel_size = 3, stride = 2,
    padding = 1, output_padding = 1),

y = tconv(x)
```

can be replaced by

```python
conv = nn.Conv2d(nic, noc, kernel_size = 3, padding = 1)

u = F.interpolate(x, scale_factor = 2, mode = 'bilinear')
y = conv(u)
```