Deep learning

7.1. Transposed convolutions

François Fleuret
https://fleuret.org/dlc/
Dec 20, 2020

Constructing deep generative architectures requires layers to increase the signal dimension, the contrary of what we have done so far with feed-forward networks.

Some generative processes optimize the input, and as such rely on back-propagation to expend the signal from a low-dimension representation to the high-dimension signal space.

The same can be done in the forward pass with **transposed convolution layers** whose forward operation corresponds to a convolution layer's backward pass.

Consider a 1d convolution with a kernel κ

$$
\begin{aligned}
y_i &= (x \circledast \kappa)_i \\
&= \sum_a x_{i+a-1}\, \kappa_a \\
&= \sum_u x_u\, \kappa_{u-i+1}.
\end{aligned}
$$

We get

$$
\begin{aligned}
\left[\frac{\partial \ell}{\partial x}\right]_u &= \frac{\partial \ell}{\partial x_u} \\
&= \sum_i \frac{\partial \ell}{\partial y_i} \frac{\partial y_i}{\partial x_u} \\
&= \sum_i \frac{\partial \ell}{\partial y_i} \kappa_{u-i+1}.
\end{aligned}
$$

which looks a lot like a standard convolution layer, except that the kernel coefficients are visited in reverse order.

This is actually the standard convolution operator from signal processing. If $*$ denotes this operation, we have

$$
(x * \kappa)_i = \sum_a x_a\, \kappa_{i-a+1}.
$$

Coming back to the backward pass of the convolution layer, if

$$
y = x \circledast \kappa
$$

then

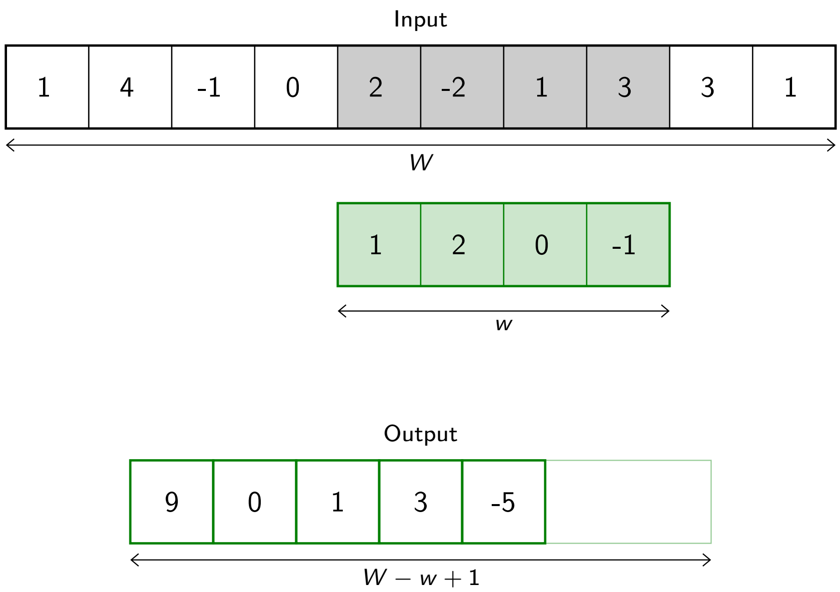
$$
\left[\frac{\partial \ell}{\partial x}\right] = \left[\frac{\partial \ell}{\partial y}\right] * \kappa.
$$

In the deep-learning field, since it corresponds to transposing the weight matrix of the equivalent fully-connected layer, it is called a **transposed convolution**.

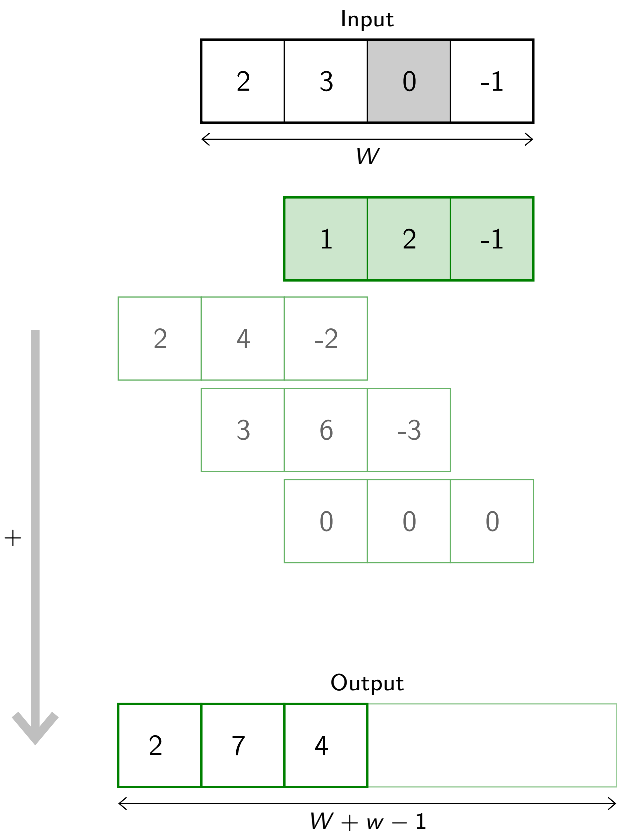
$$
\begin{pmatrix}
\kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 & 0 \\
0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 & 0 \\
0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 & 0 \\
0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3 & 0 \\
0 & 0 & 0 & 0 & \kappa_1 & \kappa_2 & \kappa_3
\end{pmatrix}^{\top}
=
\begin{pmatrix}
\kappa_1 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_1 & 0 & 0 & 0 \\
\kappa_3 & \kappa_2 & \kappa_1 & 0 & 0 \\
0 & \kappa_3 & \kappa_2 & \kappa_1 & 0 \\
0 & 0 & \kappa_3 & \kappa_2 & \kappa_1 \\
0 & 0 & 0 & \kappa_3 & \kappa_2 \\
0 & 0 & 0 & 0 & \kappa_3
\end{pmatrix}
$$

A convolution can be seen as a series of inner products, a transposed convolution can be seen as a weighted sum of translated kernels.

Convolution layer

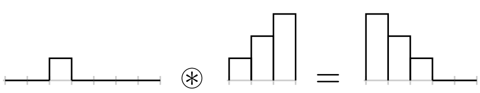


Transposed convolution layer

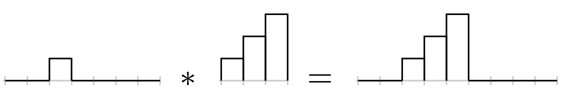


`F.conv_transpose1d` implements the operation we just described. It takes as input a batch of multi-channel samples, and produces a batch of multi-channel samples.

```
>>> x = torch.tensor([[[0., 0., 1., 0., 0., 0., 0.]]])
>>> k = torch.tensor([[[1., 2., 3.]]])
>>> F.conv1d(x, k)
tensor([[[ 3.,  2.,  1.,  0.,  0.]]])
```



```
>>> F.conv_transpose1d(x, k)
tensor([[[ 0.,  0.,  1.,  2.,  3.,  0.,  0.,  0.,  0.]]])
```



The class `nn.ConvTranspose1d` embeds that operation into a `nn.Module`.

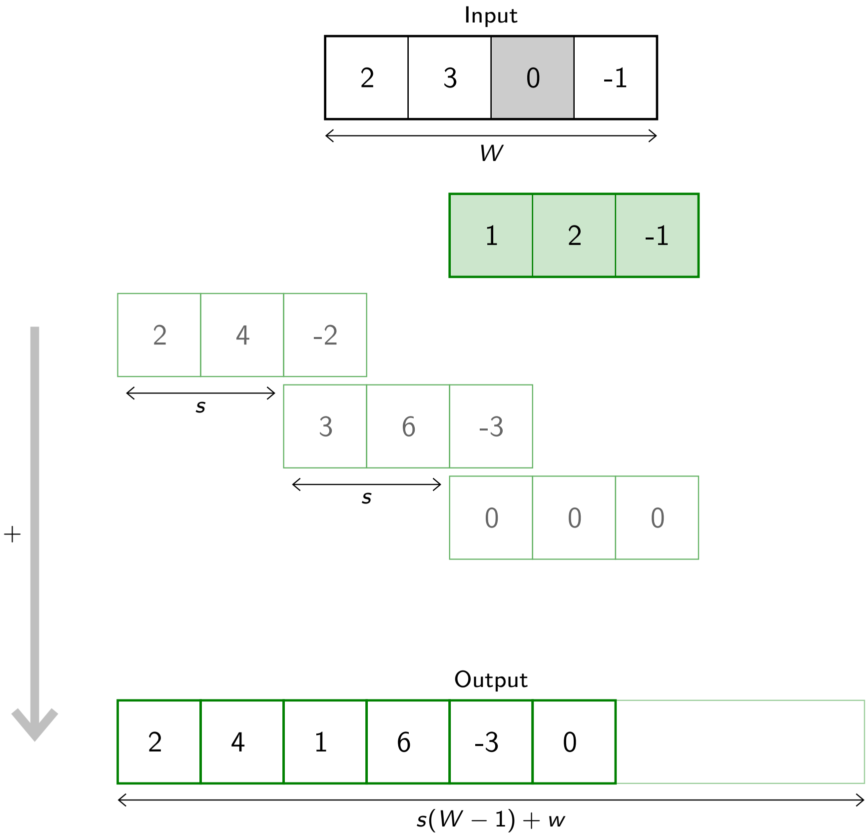
```
>>> x = torch.tensor([[[ 1., 0., 0., 0., -1.]]])
>>> m = nn.ConvTranspose1d(1, 1, kernel_size=3)
>>> with torch.autograd.no_grad():
...   m.bias.zero_()
...   m.weight.copy_(torch.tensor([ 1, 2, 1 ]))
...
Parameter containing:
tensor([0.], requires_grad=True)
Parameter containing:
tensor([[[1., 2., 1.]]], requires_grad=True)
>>> y = m(x)
>>> y
tensor([[[ 1.,  2.,  1.,  0., -1., -2., -1.]]], grad_fn=<SqueezeBackward1>)
```


Transposed convolutions also have a `dilation` parameter that behaves as for convolution and expends the kernel size without increasing the number of parameters by making it sparse.

They also have a `stride` and `padding` parameters, however, due to the relation between convolutions and transposed convolutions:

While for convolutions `stride` and `padding` are defined in the input map, for transposed convolutions these parameters are defined in the output map, and the latter modulates a cropping operation.

Transposed convolution layer (stride = 2)

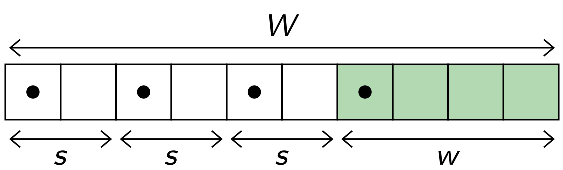


The composition of a convolution and a transposed convolution of same parameters keep the signal size [roughly] unchanged.

⚠ A convolution with a stride greater than one may ignore parts of the signal. Its composition with the corresponding transposed convolution generates a map **of the size of the observed area**.

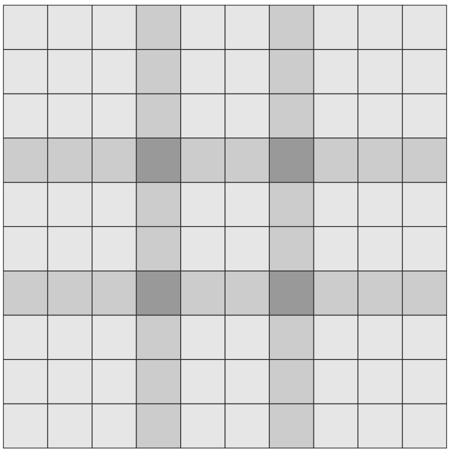
For instance, a 1d convolution of kernel size w and stride s composed with the transposed convolution of same parameters maintains the signal size W, only if

$$\exists q \in \mathbb{N}, \; W = w + s\,q.$$



It has been observed that transposed convolutions may create some grid-structure artifacts, since generated pixels are not all covered similarly.

For instance with a 4×4 kernel and stride 3



An alternative is to use an analytic up-scaling, implemented in the PyTorch functional `F.interpolate`.

```
>>> x = torch.tensor([[[[ 1., 2. ], [ 3., 4. ]]]])
>>> F.interpolate(x, scale_factor = 3, mode = 'bilinear')
tensor([[[[1.0000, 1.0000, 1.3333, 1.6667, 2.0000, 2.0000],
          [1.0000, 1.0000, 1.3333, 1.6667, 2.0000, 2.0000],
          [1.6667, 1.6667, 2.0000, 2.3333, 2.6667, 2.6667],
          [2.3333, 2.3333, 2.6667, 3.0000, 3.3333, 3.3333],
          [3.0000, 3.0000, 3.3333, 3.6667, 4.0000, 4.0000],
          [3.0000, 3.0000, 3.3333, 3.6667, 4.0000, 4.0000]]]])

>>> F.interpolate(x, scale_factor = 3, mode = 'nearest')
tensor([[[[1., 1., 1., 2., 2., 2.],
          [1., 1., 1., 2., 2., 2.],
          [1., 1., 1., 2., 2., 2.],
          [3., 3., 3., 4., 4., 4.],
          [3., 3., 3., 4., 4., 4.],
          [3., 3., 3., 4., 4., 4.]]]])
```

Such module is usually combined with a convolution to learn local corrections to undesirable artifacts of the up-scaling.

In practice, a transposed convolution such as

```
tconv = nn.ConvTranspose2d(nic, noc,
                           kernel_size = 3, stride = 2,
                           padding = 1, output_padding = 1),

y = tconv(x)
```

can be replaced by

```
conv = nn.Conv2d(nic, noc, kernel_size = 3, padding = 1)

u = F.interpolate(x, scale_factor = 2, mode = 'bilinear')
y = conv(u)
```