A first “deep” regularization technique is dropout (Srivastava et al., 2014). It consists of removing units at random during the forward pass on each sample, and putting them all back during test.

Figure 1: Dropout Neural Net Model. Left: A standard neural net with 2 hidden layers. Right: An example of a thinned net produced by applying dropout to the network on the left. Crossed units have been dropped.

(Srivastava et al., 2014)
This method increases independence between units, and distributes the representation. It generally improves performance.

“In a standard neural network, the derivative received by each parameter tells it how it should change so the final loss function is reduced, given what all other units are doing. Therefore, units may change in a way that they fix up the mistakes of the other units. This may lead to complex co-adaptations. This in turn leads to overfitting because these co-adaptations do not generalize to unseen data. **We hypothesize that for each hidden unit, dropout prevents co-adaptation by making the presence of other hidden units unreliable.** Therefore, a hidden unit cannot rely on other specific units to correct its mistakes. It must perform well in a wide variety of different contexts provided by the other hidden units.”

(Srivastava et al., 2014)

---

<table>
<thead>
<tr>
<th>Method</th>
<th>Test Classification error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>L2</td>
<td>1.62</td>
</tr>
<tr>
<td>L2 + L1 applied towards the end of training</td>
<td>1.60</td>
</tr>
<tr>
<td>L2 + KL-sparsity</td>
<td>1.55</td>
</tr>
<tr>
<td>Max-norm</td>
<td>1.35</td>
</tr>
<tr>
<td>Dropout + L2</td>
<td>1.25</td>
</tr>
<tr>
<td>Dropout + Max-norm</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 9: Comparison of different regularization methods on MNIST.

Also see how the advantages obtained from dropout vary with the probability of retaining units, size of the network and the size of the training set. These observations give some insight into why dropout works so well.

### 7.1 Effect on Features

![Figure 7](image.png)

(a) Without dropout  
(b) Dropout with \( p = 0.5 \).

Figure 7: Features learned on MNIST with one hidden layer autoencoders having 256 rectified linear units.

(Srivastava et al., 2014)

A network with dropout can be interpreted as an ensemble of \( 2^N \) models with heavy weight sharing (Goodfellow et al., 2013).
One has to decide on which units/layers to use dropout, and with what probability $p$ units are dropped.

During training, for each sample, as many Bernoulli variables as units are sampled independently to select units to remove.

Let $X$ be a unit activation, and $D$ be an independent Boolean random variable of probability $1 - p$. We have

$$E(DX) = E(D)E(X) = (1 - p)E(X)$$

To keep the means of the inputs to layers unchanged, the initial version of dropout was multiplying activations by $1 - p$ during test.

The standard variant in use is the “inverted dropout”. It multiplies activations by $\frac{1}{1-p}$ during train and keeps the network untouched during test.

Dropout is not implemented by actually switching off units, but equivalently as a module that drops activations at random on each sample.
Dropout is implemented in PyTorch as `nn.Dropout`, which is a `torch.Module`.

In the forward pass, it samples a Boolean variable for each component of the tensor it gets as input, and zeroes entries accordingly.

Default probability to drop is $p = 0.5$, but other values can be specified.

```python
>>> x = torch.full((3, 5), 1.0).requires_grad_()
>>> x
tensor([[ 1.,  1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.,  1.]])
>>> dropout = nn.Dropout(p = 0.75)
>>> y = dropout(x)
>>> y
tensor([[ 0.,  0.,  4.,  0.,  4.],
        [ 0.,  4.,  4.,  4.,  0.],
        [ 0.,  0.,  4.,  0.,  0.]])
>>> l = y.norm(2, 1).sum()
>>> l.backward()
>>> x.grad
tensor([[ 0.0000,  0.0000,  2.8284,  0.0000,  2.8284],
        [ 0.0000,  2.3094,  2.3094,  2.3094,  0.0000],
        [ 0.0000,  0.0000,  4.0000,  0.0000,  0.0000]])
```
If we have a network

```python
model = nn.Sequential(nn.Linear(10, 100), nn.ReLU(),
                      nn.Linear(100, 50), nn.ReLU(),
                      nn.Linear(50, 2));
```

we can simply add dropout layers

```python
model = nn.Sequential(nn.Linear(10, 100), nn.ReLU(),
                      nn.Dropout(),
                      nn.Linear(100, 50), nn.ReLU(),
                      nn.Dropout(),
                      nn.Linear(50, 2));
```

⚠️ A model using dropout has to be set in “train” or “test” mode.

The method `nn.Module.train(mode)` recursively sets the flag `training` to all sub-modules.

```python
>>> dropout = nn.Dropout()
>>> model = nn.Sequential(nn.Linear(3, 10), dropout, nn.Linear(10, 3))
>>> dropout.training
True
>>> model.train(False)
Sequential (
  (0): Linear (3 -> 10)
  (1): Dropout (p = 0.5)
  (2): Linear (10 -> 3)
)
>>> dropout.training
False
```
As pointed out by Tompson et al. (2015), units in a 2d activation map are generally locally correlated, and dropout has virtually no effect. They proposed SpatialDropout, which drops channels instead of individual units.

```python
>>> dropout2d = nn.Dropout2d()
>>> x = torch.full((2, 3, 2, 4), 1.)
>>> dropout2d(x)
tensor([[[ 2., 2., 2., 2.], 
            [ 0., 0., 0., 0.]],
            [[ 2., 2., 2., 2.], 
            [ 0., 0., 0., 0.]],
            [[ 2., 2., 2., 2.], 
            [ 2., 2., 2., 2.]]],

[[[ 2., 2., 2., 2.], 
    [ 2., 2., 2., 2.]],
[[ 0., 0., 0., 0.], 
    [ 0., 0., 0., 0.]],
[[ 0., 0., 0., 0.], 
    [ 0., 0., 0., 0.]]])
```

Another variant is dropconnect, which drops connections instead of units.

Figure 1. (a): An example model layout for a single DropConnect layer. After running feature extractor $g()$ on input $x$, a random instantiation of the mask $M$ (e.g. (b)), masks out the weight matrix $W$. The masked weights are multiplied with this feature vector to produce $u$ which is the input to an activation function $a$ and a softmax layer $s$. For comparison, (c) shows an effective weight mask for elements that DropConnect uses when applied to the previous layer’s output (red columns) and this layer’s output (green rows). Note the lack of structure in (b) compared to (c).

(Wan et al., 2013)

It cannot be implemented as a separate layer and is computationally intensive.
References


