Deep learning

12.3. Word embeddings and translation

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Word embeddings and CBOW

An important application domain for machine intelligence is Natural Language Processing (NLP).

- Speech and (hand)writing recognition,
- translation,
- question answering.
- part-of-speech tagging,
- sentiment analysis,
- auto-captioning.

While language modeling was historically addressed with formal methods, in particular generative grammars, state-of-the-art and deployed methods are now heavily based on statistical learning and deep learning.

A core difficulty of Natural Language Processing is to devise a proper density model for sequences of words.

Since a vocabulary is usually of the order of $10^4 - 10^6$ words, empirical distributions can not be estimated for more than triplets of words.

The standard strategy to mitigate this problem is to embed words into a geometrical space and exploit regularities for further [statistical] modeling.

The geometry after embedding should account for synonymy, but also for identical word classes, etc. *E.g.* we would like such an embedding to make "cat" and "tiger" close, but also "red" and "blue", or "eat" and "work", etc.

Even though they are not "deep", classical word embedding models are key elements of NLP with deep-learning.

Let

$$k_t \in \{1, \dots, W\}, \; t = 1, \dots, T$$

be a training sequence of T words, encoded as IDs from a W words vocabulary.

Given an embedding dimension D, the objective is to learn vectors

$$E_k \in \mathbb{R}^D, \; k \in \{1, \dots, W\}$$

so that "similar" words are embedded with "similar" vectors.

A common word embedding is the Continuous Bag of Words (CBOW) version of word2vec (Mikolov et al., 2013a).

In this model, the embedding vectors are chosen so that a word can be [linearly] predicted from the sum of the embeddings of words around it.

More formally, let $C \in \mathbb{N}^*$ be a "context size", and

$$\mathscr{C}_t = (k_{t-C}, \dots, k_{t-1}, k_{t+1}, \dots, k_{t+C})$$

be the "context" around k_t, that is the indexes of words around it.

$$k_1 \quad \cdots \quad \underbrace{\overbrace{k_{t-C} \quad \cdots \quad k_{t-1}}^{C} \quad k_t \quad \overbrace{k_{t+1} \quad \cdots \quad k_{t+C}}^{C}}_{\mathscr{C}_t} \quad \cdots \quad k_T$$

The embeddings vectors

$$E_k \in \mathbb{R}^D,\ k = 1, \dots, W,$$

are optimized jointly with an array

$$M \in \mathbb{R}^{W \times D}$$

so that the predicted vector of W scores

$$\psi(t) = M \sum_{k \in \mathscr{C}_t} E_k$$

is a good predictor of the value of k_t.

Ideally we would minimize the cross-entropy between the vector of scores $\psi(t) \in \mathbb{R}^W$ and the class k_t

$$\sum_t -\log\left(\frac{\exp \psi(t)_{k_t}}{\sum_{k=1}^{W} \exp \psi(t)_k}\right).$$

However, given the vocabulary size, doing so is numerically unstable and computationally demanding.

The “negative sampling” approach uses the prediction for the correct class k_t and only $Q \ll W$ incorrect classes $\kappa_{t,1}, \dots, \kappa_{t,Q}$ sampled at random.

In our implementation we take the later uniformly in $\{1, \dots, W\}$ and use the same loss as Mikolov et al. (2013b):

$$\sum_t \log\left(1 + e^{-\psi(t)_{k_t}}\right) + \sum_{q=1}^{Q} \log\left(1 + e^{\psi(t)_{\kappa_{t,q}}}\right).$$

We want $\psi(t)_{k_t}$ to be large and all the $\psi(t)_{\kappa_{t,q}}$ to be small.

Although the operation

$$x \mapsto E_x$$

could be implemented as the product between a one-hot vector and a matrix, it is far more efficient to use an actual lookup table.

The PyTorch module `nn.Embedding` does precisely that. It is parametrized with a number N of words to embed, and an embedding dimension D.

It gets as input an integer tensor of arbitrary dimension $A_1 \times \cdots \times A_U$, containing values in $\{0, \dots, N-1\}$ and it returns a float tensor of dimension $A_1 \times \cdots \times A_U \times D$.

If w are the embedding vectors, x the input tensor, y the result, we have

$$y[a_1, \dots, a_U, d] = w[x[a_1, \dots, a_U]][d].$$


```
>>> e = nn.Embedding(num_embeddings = 10, embedding_dim = 3)
>>> x = torch.tensor([[1, 1, 2, 2], [0, 1, 9, 9]])
>>> y = e(x)
>>> y.size()
torch.Size([2, 4, 3])
>>> y
tensor([[[ 0.1310,  0.7812, -0.8920],
         [ 0.1310,  0.7812, -0.8920],
         [ 0.9378, -2.5204,  0.3979],
         [ 0.9378, -2.5204,  0.3979]],

        [[-1.5423, -0.1116,  0.3108],
         [ 0.1310,  0.7812, -0.8920],
         [-0.6145, -0.2464,  0.7647],
         [-0.6145, -0.2464,  0.7647]]], grad_fn=<EmbeddingBackward>)
```

Our CBOW model has as parameters two embeddings

$$E \in \mathbb{R}^{W \times D} \quad \text{and} \quad M \in \mathbb{R}^{W \times D}.$$

Its `forward` gets as input a pair of integer tensors corresponding to a batch of size B:

- c of size $B \times 2C$ contains the IDs of the words in a context, and
- d of size $B \times R$ contains the IDs, for each of the B contexts, of the R words for which we want the prediction score (that will be the correct one and Q negative ones).

it returns a tensor y of size $B \times R$ containing the dot products.

$$y[n,j] = \frac{1}{D} M_{d[n,j]} \cdot \left(\sum_i E_{c[n,i]} \right).$$


```
class CBOW(nn.Module):
    def __init__(self, voc_size = 0, embed_dim = 0):
        super().__init__()
        self.embed_dim = embed_dim
        self.embed_E = nn.Embedding(voc_size, embed_dim)
        self.embed_M = nn.Embedding(voc_size, embed_dim)

    def forward(self, c, d):
        sum_w_E = self.embed_E(c).sum(1, keepdim = True).transpose(1, 2)
        w_M = self.embed_M(d)
        return w_M.bmm(sum_w_E).squeeze(2) / self.embed_dim
```

Regarding the loss, we can use `nn.BCEWithLogitsLoss` which implements

$$\sum_t y_t \log(1 + \exp(-x_t)) + (1 - y_t) \log(1 + \exp(x_t)).$$

It takes care in particular of the numerical problem that may arise for large values of x_t if implemented "naively".

Before training a model, we need to prepare data tensors of word IDs from a text file. We will use a 100Mb text file taken from Wikipedia and

- make it lower-cap,
- remove all non-letter characters,
- replace all words that appear less than 100 times with `'*'`,
- associate to each word a unique id.

From the resulting sequence of length T stored in a integer tensor, and the context size C, we will generate mini-batches, each of two tensors

- a 'context' integer tensor c of dimension $B \times 2C$, and
- a 'word' integer tensor w of dimension B.

If the corpus is “The black cat plays with the black ball.”, we will get the following word IDs:

`the`: 0, `black`: 1, `cat` : 2, `plays`: 3, `with`: 4, `ball`: 5.

The corpus will be encoded as

the	black	cat	plays	with	the	black	ball
0	1	2	3	4	0	1	5

and the data and label tensors will be

Words					IDs					c	w
the	black	cat	plays	with	0	1	2	3	4	$0, 1, 3, 4$	2
black	cat	plays	with	the	1	2	3	4	0	$1, 2, 4, 0$	3
cat	plays	with	the	black	2	3	4	0	1	$2, 3, 0, 1$	4
plays	with	the	black	ball	3	4	0	1	5	$3, 4, 1, 5$	0

We can train the model for an epoch with:

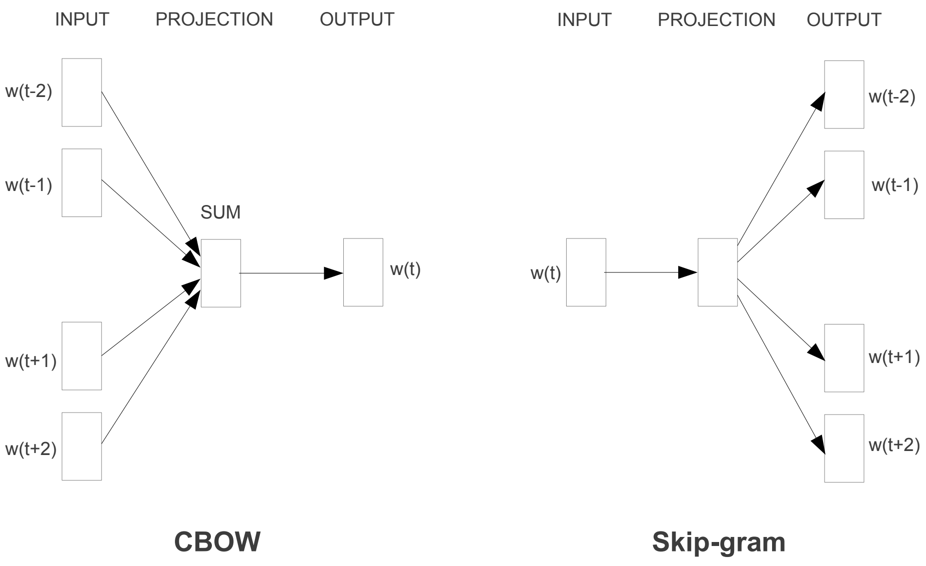
```
for k in range(0, id_seq.size(0) - 2 * context_size - batch_size, batch_size):
    c, w = extract_batch(id_seq, k, batch_size, context_size)

    d = torch.randint(voc_size, (w.size(0), 1 + nb_neg_samples))
    d[:, 0] = w

    target = torch.zeros(d.size())
    target[:, 0] = 1.0

    output = model(c, d)
    loss = bce_loss(output, target)

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

As results, some nearest-neighbors for the cosine distance between the embeddings

$$d(w, w') = \frac{E_w \cdot E_{w'}}{\|E_w\| \|E_{w'}\|}.$$

	paris		bike		cat		fortress		powerful
0.61	parisian	0.61	bicycle	0.55	cats	0.61	fortresses	0.47	formidable
0.59	france	0.51	bicycles	0.54	dog	0.55	citadel	0.44	power
0.55	brussels	0.51	bikes	0.49	kitten	0.55	castle	0.44	potent
0.53	bordeaux	0.49	biking	0.44	feline	0.52	fortifications	0.40	fearsome
0.51	toulouse	0.47	motorcycle	0.42	pet	0.51	forts	0.40	destroy
0.51	vienna	0.43	cyclists	0.40	dogs	0.50	siege	0.39	wielded
0.51	strasbourg	0.42	riders	0.40	kittens	0.49	stronghold	0.38	versatile
0.49	munich	0.41	sled	0.39	hound	0.49	castles	0.38	capable
0.49	marseille	0.41	triathlon	0.39	squirrel	0.48	monastery	0.38	strongest
0.48	rouen	0.41	car	0.38	mouse	0.48	besieged	0.37	able

An alternative algorithm is the skip-gram model, which optimizes the embedding so that a word can be predicted by any individual word in its context (Mikolov et al., 2013a).



(Mikolov et al., 2013a)

Trained on large corpora, such models reflect semantic relations in the linear structure of the embedding space. *E.g.*

$$E_{[paris]} - E_{[france]} + E_{[italy]} \simeq E_{[rome]}$$

Table 8: *Examples of the word pair relationships, using the best word vectors from Table 4 (Skip-gram model trained on 783M words with 300 dimensionality).*

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza

(Mikolov et al., 2013a)

The main benefit of word embeddings is that they are trained with unsupervised corpora, hence possibly extremely large.

This modeling can then be leveraged for small-corpora tasks such as

- sentiment analysis,
- question answering,
- topic classification,
- etc.

Sequence-to-sequence translation

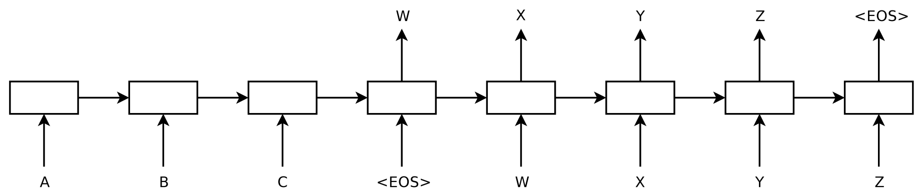


Figure 1: Our model reads an input sentence “ABC” and produces “WXYZ” as the output sentence. The model stops making predictions after outputting the end-of-sentence token. Note that the LSTM reads the input sentence in reverse, because doing so introduces many short term dependencies in the data that make the optimization problem much easier.

(Sutskever et al., 2014)

English to French translation.

Training:

- corpus 12M sentences, 348M French words, 304M English words,
- LSTM with 4 layers, one for encoding, one for decoding,
- $160,000$ words input vocabulary, $80,000$ output vocabulary,
- $1,000$ dimensions word embedding, 384M parameters total,
- input sentence is reversed,
- gradient clipping.

The hidden state that contains the information to generate the translation is of dimension $8,000$.

Inference is done with a "beam search", that consists of greedily increasing the size of the predicted sequence while keeping a bag of K best ones.

Comparing a produced sentence to a reference one is complex, since it is related to their semantic content.

A widely used measure is the BLEU score, that counts the fraction of groups of one, two, three and four words (aka "n-grams") from the generated sentence that appear in the reference translations (Papineni et al., 2002).

The exact definition is complex, and the validity of this score is disputable since it poorly accounts for semantic.

Method	test BLEU score (ntst14)
Bahdanau et al. [2]	28.45
Baseline System [29]	33.30
Single forward LSTM, beam size 12	26.17
Single reversed LSTM, beam size 12	30.59
Ensemble of 5 reversed LSTMs, beam size 1	33.00
Ensemble of 2 reversed LSTMs, beam size 12	33.27
Ensemble of 5 reversed LSTMs, beam size 2	34.50
Ensemble of 5 reversed LSTMs, beam size 12	**34.81**

Table 1: The performance of the LSTM on WMT'14 English to French test set (ntst14). Note that an ensemble of 5 LSTMs with a beam of size 2 is cheaper than of a single LSTM with a beam of size 12.

(Sutskever et al., 2014)

Type	Sentence
Our model	Ulrich UNK , membre du conseil d' administration du constructeur automobile Audi , affirme qu' il s' agit d' une pratique courante depuis des années pour que les téléphones portables puissent être collectés avant les réunions du conseil d' administration afin qu' ils ne soient pas utilisés comme appareils d' écoute à distance .
Truth	Ulrich Hackenberg , membre du conseil d' administration du constructeur automobile Audi , déclare que la collecte des téléphones portables avant les réunions du conseil , afin qu' ils ne puissent pas être utilisés comme appareils d' écoute à distance , est une pratique courante depuis des années .
Our model	" Les téléphones cellulaires , qui sont vraiment une question , non seulement parce qu' ils pourraient potentiellement causer des interférences avec les appareils de navigation , mais nous savons , selon la FCC , qu' ils pourraient interférer avec les tours de téléphone cellulaire lorsqu' ils sont dans l' air " , dit UNK .
Truth	" Les téléphones portables sont véritablement un problème , non seulement parce qu' ils pourraient éventuellement créer des interférences avec les instruments de navigation , mais parce que nous savons , d' après la FCC , qu' ils pourraient perturber les antennes-relais de téléphonie mobile s' ils sont utilisés à bord " , a déclaré Rosenker .
Our model	Avec la crémation , il y a un " sentiment de violence contre le corps d' un être cher " , qui sera " réduit à une pile de cendres " en très peu de temps au lieu d' un processus de décomposition " qui accompagnera les étapes du deuil " .
Truth	Il y a , avec la crémation , " une violence faite au corps aimé " , qui va être " réduit à un tas de cendres " en très peu de temps , et non après un processus de décomposition , qui " accompagnerait les phases du deuil " .

Table 3: A few examples of long translations produced by the LSTM alongside the ground truth translations. The reader can verify that the translations are sensible using Google translate.

(Sutskever et al., 2014)

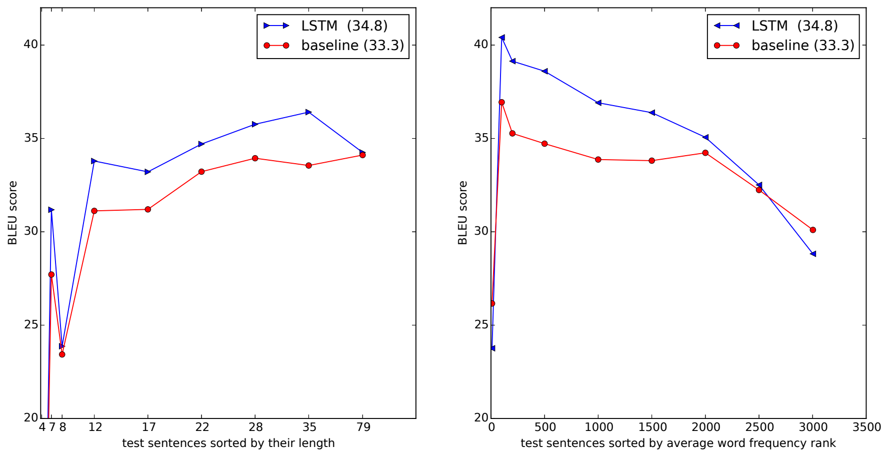


Figure 3: The left plot shows the performance of our system as a function of sentence length, where the x-axis corresponds to the test sentences sorted by their length and is marked by the actual sequence lengths. There is no degradation on sentences with less than 35 words, there is only a minor degradation on the longest sentences. The right plot shows the LSTM's performance on sentences with progressively more rare words, where the x-axis corresponds to the test sentences sorted by their "average word frequency rank".

(Sutskever et al., 2014)

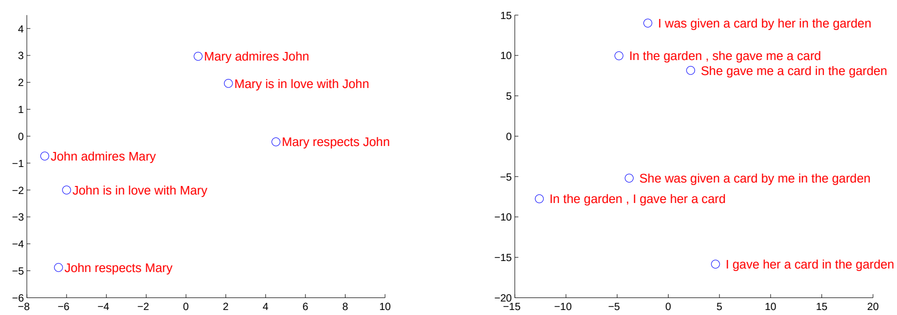


Figure 2: The figure shows a 2-dimensional PCA projection of the LSTM hidden states that are obtained after processing the phrases in the figures. The phrases are clustered by meaning, which in these examples is primarily a function of word order, which would be difficult to capture with a bag-of-words model. Notice that both clusters have similar internal structure.

(Sutskever et al., 2014)

References

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K. Papineni, S. Roukos, T. Ward, and W.-J. Zhu. **Bleu: A method for automatic evaluation of machine translation**. In Proceedings of the Annual Meeting of the Association for Computational Linguistics (ACL), pages 311–318. Association for Computational Linguistics, 2002.

I. Sutskever, O. Vinyals, and Q. V. Le. **Sequence to sequence learning with neural networks**. In Neural Information Processing Systems (NIPS), pages 3104–3112, 2014.