Deep learning

1.5. High dimension tensors

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A tensor can be of several types:

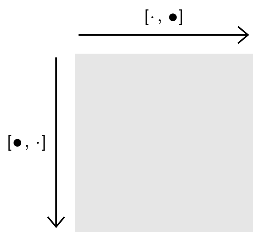
- `torch.float16`, `torch.float32`, `torch.float64`,
- `torch.uint8`,
- `torch.int8`, `torch.int16`, `torch.int32`, `torch.int64`

and can be located either in the CPU's or in a GPU's memory.

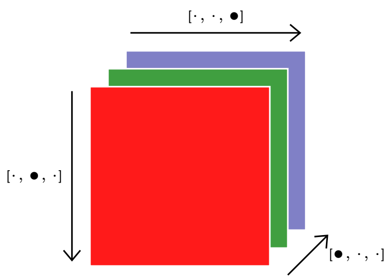
Operations with tensors stored in a certain device's memory are done by that device. We will come back to that later.

```
>>> x = torch.zeros(1, 3)
>>> x.dtype, x.device
(torch.float32, device(type='cpu'))
>>> x = x.long()
>>> x.dtype, x.device
(torch.int64, device(type='cpu'))
>>> x = x.to('cuda')
>>> x.dtype, x.device
(torch.int64, device(type='cuda', index=0))
```

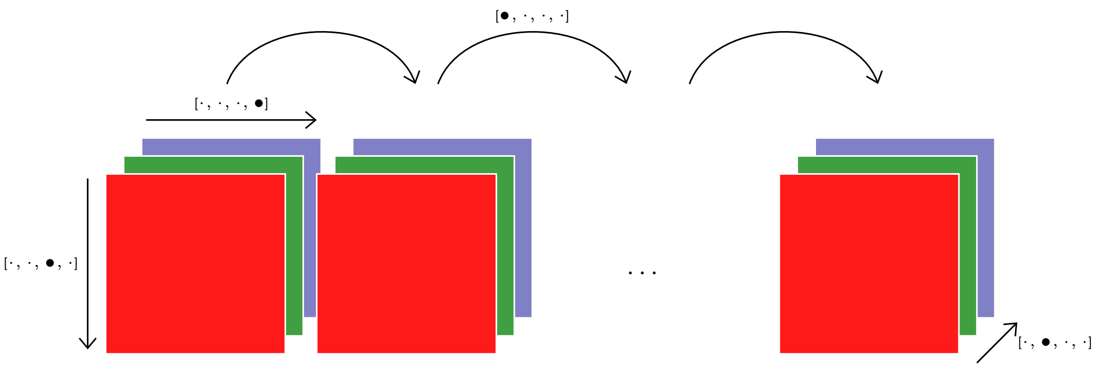

2d tensor (*e.g.* grayscale image)



3d tensor (*e.g.* rgb image)



4d tensor (*e.g.* sequence of rgb images)



Here are a few examples from the immense library of tensor operations:

Creation

- `torch.empty(*size, ...)`
- `torch.zeros(*size, ...)`
- `torch.full(size, value, ...)`
- `torch.tensor(sequence, ...)`
- `torch.eye(n, ...)`
- `torch.from_numpy(ndarray)`

Indexing, Slicing, Joining, Mutating

- `torch.Tensor.view(*size)`
- `torch.cat(inputs, dimension=0)`
- `torch.chunk(tensor, nb_chunks, dim=0)[source]`
- `torch.split(tensor, split_size, dim=0)[source]`
- `torch.index_select(input, dim, index, out=None)`
- `torch.t(input, out=None)`
- `torch.transpose(input, dim0, dim1, out=None)`

Filling

- `Tensor.fill_(value)`
- `torch.bernoulli_(proba)`
- `torch.normal_([mu, [std]])`

Pointwise math

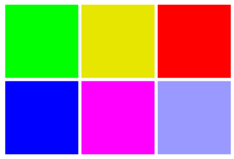
- `torch.abs(input, out=None)`
- `torch.add()`
- `torch.cos(input, out=None)`
- `torch.sigmoid(input, out=None)`

Math reduction

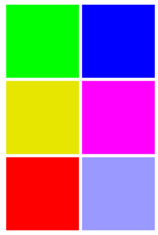
- `torch.dist(input, other, p=2, out=None)`
- `torch.mean()`
- `torch.norm()`
- `torch.std()`
- `torch.sum()`

BLAS and LAPACK Operations

- `torch.eig(a, eigenvectors=False, out=None)`
- `torch.lstsq(B, A, out=None)`
- `torch.inverse(input, out=None)`
- `torch.mm(mat1, mat2, out=None)`
- `torch.mv(mat, vec, out=None)`

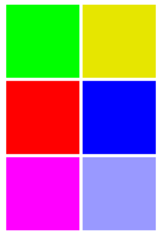


```
x = torch.tensor([ [ 1, 3, 0 ],
                   [ 2, 4, 6 ] ])
```

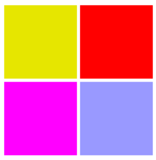


`x.t()`

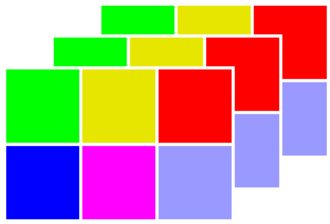
`x.view(-1)`



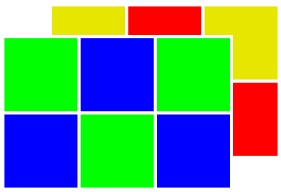
`x.view(3, -1)`



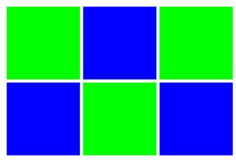
`x[:, 1:3]`



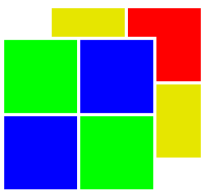
`x.view(1, 2, 3).expand(3, 2, 3)`



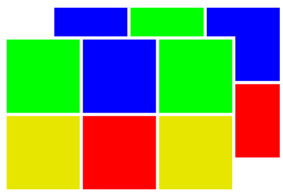
```
x = torch.tensor([ [ [ 1, 2, 1 ],
                     [ 2, 1, 2 ] ],
                   [ [ 3, 0, 3 ],
                     [ 0, 3, 0 ] ] ])
```



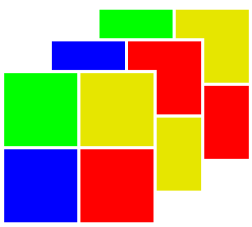
`x[0:1, :, :]`



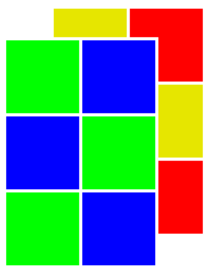
`x[:, :, 0:2]`



`x.transpose(0, 1)`



`x.transpose(0, 2)`



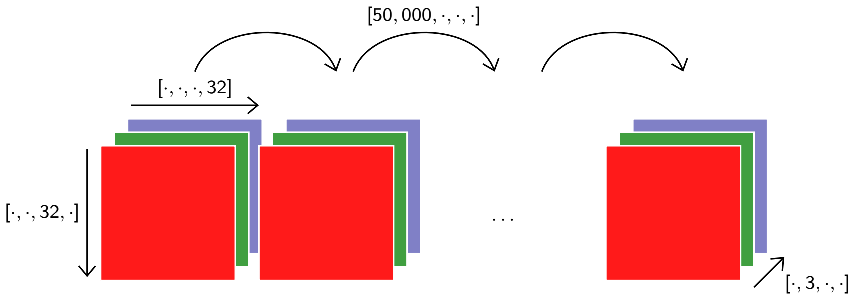
`x.transpose(1, 2)`

PyTorch offers simple interfaces to standard image data-bases.

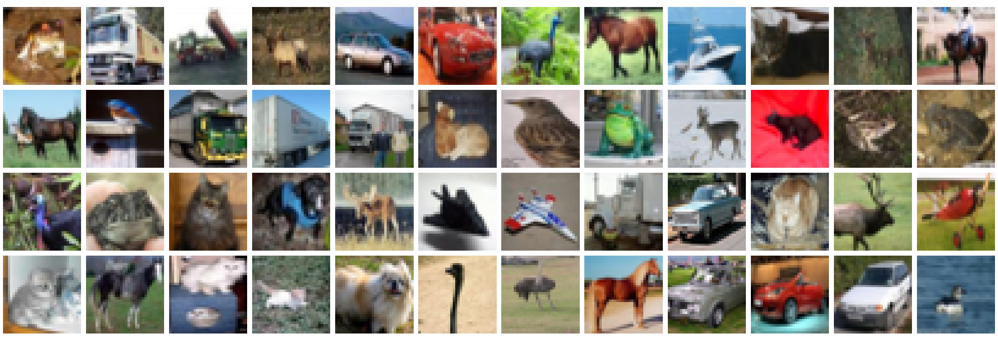
```
import torch, torchvision
cifar = torchvision.datasets.CIFAR10('./cifar10/', train = True, download = True)
x = torch.from_numpy(cifar.data).permute(0, 3, 1, 2).float() / 255
print(x.dtype, x.size(), x.min().item(), x.max().item())
```

prints

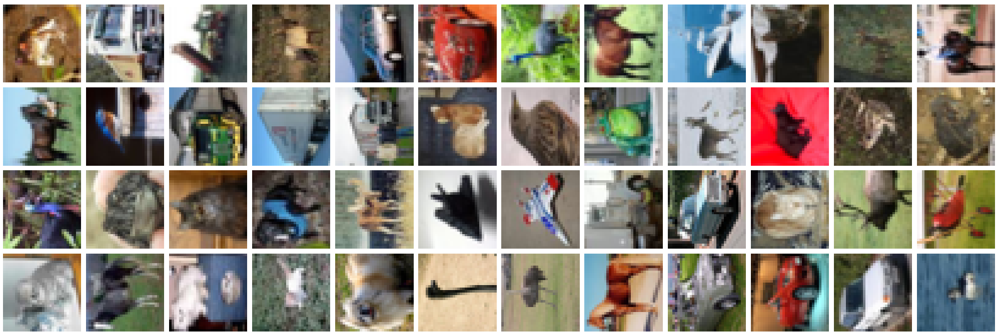
```
Files already downloaded and verified
torch.float32 torch.Size([50000, 3, 32, 32]) 0.0 1.0
```



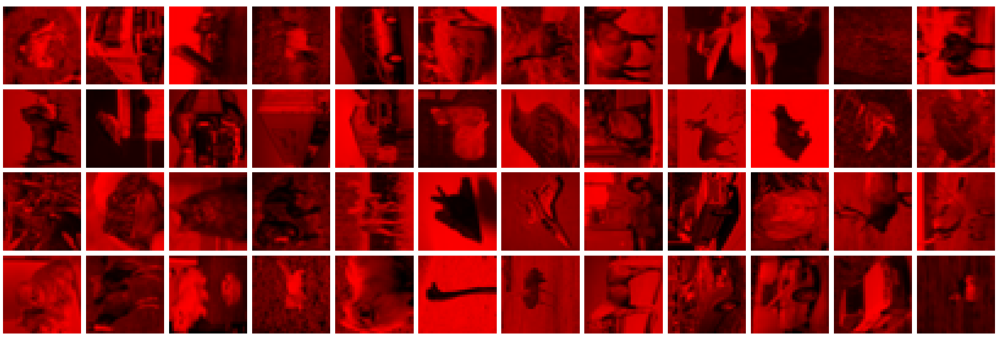

```
# Narrows to the first images, converts to float
x = x[:48]

# Saves these samples as a single image
torchvision.utils.save_image(x, 'cifar-4x12.png',
                             nrow = 12, pad_value = 1.0)
```



```
# Switches the row and column indexes
x.transpose_(2, 3)
torchvision.utils.save_image(x, 'cifar-4x12-rotated.png',
                             nrow = 12, pad_value = 1.0)
```




```
# Kills the green and blue channels
x[:, 1:3].fill_(0)
torchvision.utils.save_image(x, 'cifar-4x12-rotated-and-red.png',
                             nrow = 12, pad_value = 1.0)
```



Broadcasting and Einstein summations

Broadcasting automagically expands dimensions by replicating coefficients, when it is necessary to perform operations that are "intuitively reasonable".

For instance:

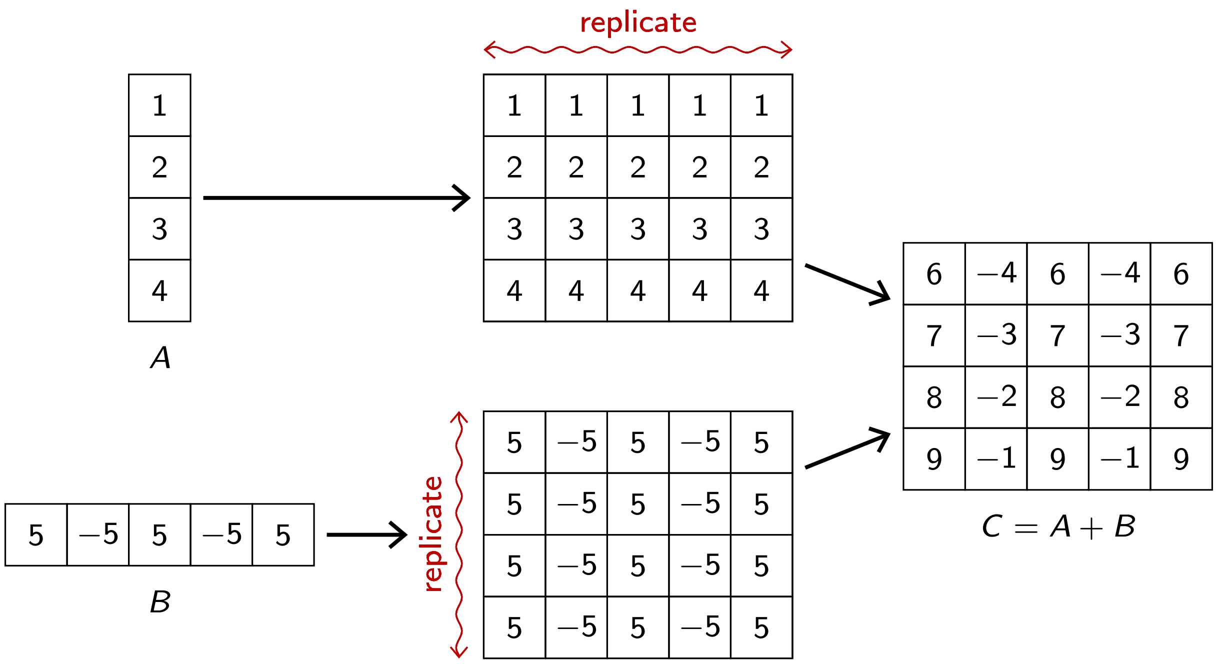
```
>>> x = torch.empty(100, 4).normal_(2)
>>> x.mean(0)
tensor([2.0476, 2.0133, 1.9109, 1.8588])
>>> x -= x.mean(0) # This should not work, but it does!
>>> x.mean(0)
tensor([-4.0531e-08, -4.4703e-07, -1.3471e-07,  3.5763e-09])
```

Precisely, broadcasting proceeds as follows:

1. If one of the tensors has fewer dimensions than the other, it is reshaped by adding as many dimensions of size 1 as necessary in the front; then
2. for every dimension mismatch, **if one of the two tensors is of size one,** it is expanded along this axis by replicating coefficients.

If there is a tensor size mismatch for one of the dimension and neither of them is one, the operation fails.


```
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]])
C = A + B
```



Broadcasted

A powerful generic tool for complex tensorial operations is the **Einstein summation convention**. It provides a concise way of describing dimension re-ordering and summing of component-wise products along some of them.

`torch.einsum` takes as argument a string describing the operation, the tensors to operate on, and returns a tensor.

The operation string is a comma-separated list of indexing, followed by the indexing for the result.

Summations are executed on all indexes not appearing in the result indexing.

For instance, we can formulate that way the standard matrix product:

$$\mathbb{R}^{A \times B} \times \mathbb{R}^{B \times C} \rightarrow \mathbb{R}^{A \times C}$$

$$\forall i, k, \; m_{i,k} = \sum_j p_{i,j} q_{j,k}$$

```
m = torch.einsum('ij,jk->ik', p, q)
```

The summation is done along `j` since it does not appear after the `->`.

```
>>> p = torch.rand(2, 5)
>>> q = torch.rand(5, 4)
>>> torch.einsum('ij,jk->ik', p, q)
tensor([[2.0833, 1.1046, 1.5220, 0.4405],
        [2.1338, 1.2601, 1.4226, 0.8641]])
>>> p@q
tensor([[2.0833, 1.1046, 1.5220, 0.4405],
        [2.1338, 1.2601, 1.4226, 0.8641]])
```

Matrix-vector product:

$$\mathbb{R}^{A\times B} \times \mathbb{R}^{B} \to \mathbb{R}^{A}$$
$$\forall i,k,\ w_i = \sum_j m_{i,j} v_j$$

```
w = torch.einsum('ij,j->i', m, v)
```

Hadamard (component-wise) product:

$$\mathbb{R}^{A\times B} \times \mathbb{R}^{A\times B} \to \mathbb{R}^{A\times B}$$
$$\forall i,j,\ m_{i,j} = p_{i,j} q_{i,j}$$

```
m = torch.einsum('ij,ij->ij', p, q)
```

Extracting the diagonal:

$$\mathbb{R}^{D\times D} \to \mathbb{R}^{D}$$
$$\forall i,k,\ v_i = m_{i,i}$$

```
v = torch.einsum('ii->i', m)
```


Batch matrix product:

$$\mathbb{R}^{N\times A\times B} \times \mathbb{R}^{N\times B\times C} \to \mathbb{R}^{N\times A\times C}$$
$$\forall n,i,k,\ m_{n,i,k} = \sum_j p_{n,i,j} q_{n,j,k}$$

```
m = torch.einsum('nij,njk->nik', p, q)
```

Batch trace:

$$\mathbb{R}^{N\times D\times D} \to \mathbb{R}^{N}$$
$$\forall n,\ t_n = \sum_i m_{n,i,i}$$

```
t = torch.einsum('nii->n', m)
```

Tri-linear product along a channel:

$$\mathbb{R}^{N\times C\times T} \times \mathbb{R}^{N\times C\times T} \times \mathbb{R}^{N\times C\times T} \to \mathbb{R}^{N\times T}$$
$$\forall n,t,\ m_{n,t} = \sum_c p_{n,c,t} q_{n,c,t} r_{n,c,t}$$

```
m = torch.einsum('nct,nct,nct->nt', p, q, r)
```