Deep learning

1.4. Tensor basics and linear regression

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A tensor is a generalized matrix, a finite table of numerical values indexed along several discrete dimensions.

- A 0d tensor is a scalar,
- A 1d tensor is a vector (*e.g.* a sound sample),
- A 2d tensor is a matrix (*e.g.* a grayscale image),
- A 3d tensor can be seen as a vector of identically sized matrix (*e.g.* a multi-channel image),
- A 4d tensor can be seen as a matrix of identically sized matrices, or a sequence of 3d tensors (*e.g.* a sequence of multi-channel images),
- etc.

Tensors are used to encode the signal to process, but also the internal states and parameters of models.

Manipulating data through this constrained structure allows to use CPUs and GPUs at [near] peak performance.

Compounded data structures can represent more diverse data types.

PyTorch's main features are:

- Efficient tensor operations on CPU/GPU,
- automatic on-the-fly differentiation (autograd),
- optimizers,
- data I/O.

"Efficient tensor operations" encompass both standard linear algebra and, as we will see later, deep-learning specific operations (convolution, pooling, etc.)

A key specificity of PyTorch is the central role of autograd to compute derivatives of *anything* ! We will come back to this.


```
>>> x = torch.empty(2, 5)
>>> x.size()
torch.Size([2, 5])
>>> x.fill_(1.125)
tensor([[ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250],
        [ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250]])
>>> x.mean()
tensor(1.1250)
>>> x.std()
tensor(0.)
>>> x.sum()
tensor(11.2500)
>>> x.sum().item()
11.25
```

In-place operations are suffixed with an underscore, and a 0d tensor can be converted back to a Python scalar with `item()`.

⚠ Reading a coefficient also generates a 0d tensor.

```
>>> x = torch.tensor([[11., 12., 13.], [21., 22., 23.]])
>>> x[1, 2]
tensor(23.)
```

PyTorch provides operators for component-wise and vector/matrix operations.

```
>>> x = torch.tensor([ 10., 20., 30.])
>>> y = torch.tensor([ 11., 21., 31.])
>>> x + y
tensor([ 21.,  41.,  61.])
>>> x * y
tensor([ 110.,  420.,  930.])
>>> x**2
tensor([ 100.,  400.,  900.])
>>> m = torch.tensor([[ 0., 0., 3. ],
...                   [ 0., 2., 0. ],
...                   [ 1., 0., 0. ]])
>>> m.mv(x)
tensor([ 90.,  40.,  10.])
>>> m @ x
tensor([ 90.,  40.,  10.])
```


And as in Numpy, the : symbol defines a range of values for an index and allows to slice tensors.

```
>>> import torch
>>> x = torch.empty(2, 4).random_(10)
>>> x
tensor([[8., 1., 1., 3.],
        [7., 0., 7., 5.]])
>>> x[0]
tensor([8., 1., 1., 3.])
>>> x[0, :]
tensor([8., 1., 1., 3.])
>>> x[:, 0]
tensor([8., 7.])
>>> x[:, 1:3] = -1
>>> x
tensor([[ 8., -1., -1.,  3.],
        [ 7., -1., -1.,  5.]])
```

PyTorch provides interfacing to standard linear operations, such as linear system solving or Eigen-decomposition.

```
>>> y = torch.empty(3).normal_()
>>> y
tensor([ 0.0477,  0.8834, -1.5996])
>>> m = torch.empty(3, 3).normal_()
>>> q, _ = torch.lstsq(y, m)
>>> torch.mm(m, q)
tensor([[ 0.0477],
        [ 0.8834],
        [-1.5996]])
```


Example: linear regression

Given a list of points

$$(x_n, y_n) \in \mathbb{R} \times \mathbb{R}, \ n = 1, \dots, N,$$

can we find the "best line"

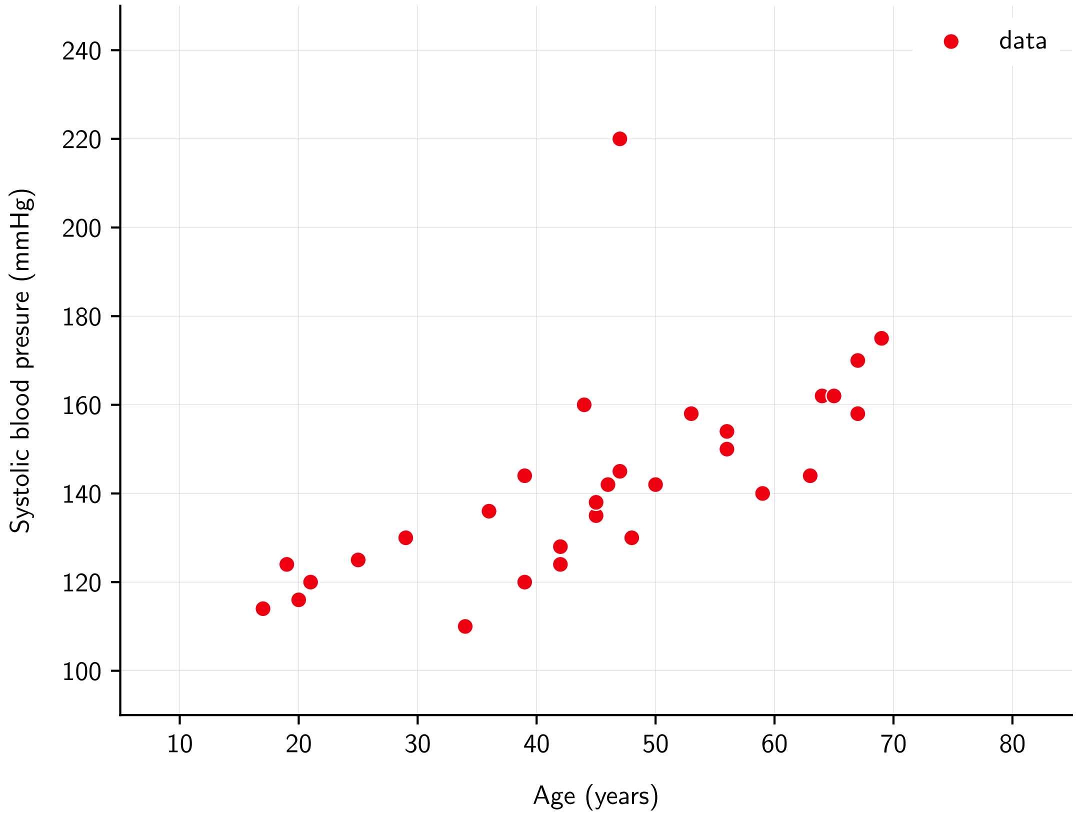
$$f(x; a, b) = ax + b$$

going "through the points", *e.g.* minimizing the mean square error

$$\underset{a,b}{\operatorname{argmin}} \frac{1}{N} \sum_{n=1}^{N} \big(\underbrace{ax_n + b}_{f(x_n; a, b)} - y_n \big)^2.$$

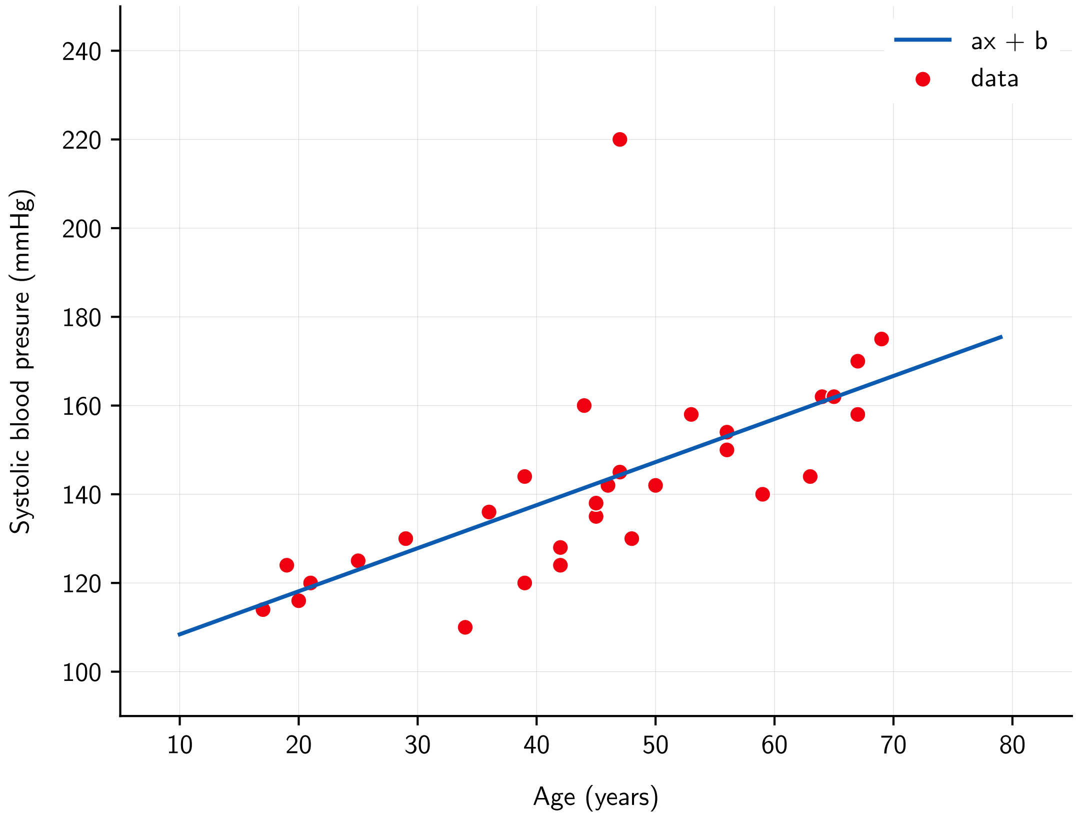
Such a model would allow to predict the y associated to a new x, simply by calculating $f(x; a, b)$.


```
bash> cat systolic-blood-pressure-vs-age.dat
39  144
47  220
45  138
47  145
65  162
46  142
67  170
42  124
67  158
56  154
64  162
56  150
59  140
34  110
42  128
/.../
```



$$\underbrace{\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_N & y_N \end{pmatrix}}_{\text{data} \in \mathbb{R}^{N \times 2}} \qquad \underbrace{\begin{pmatrix} x_1 & 1.0 \\ x_2 & 1.0 \\ \vdots & \vdots \\ x_N & 1.0 \end{pmatrix}}_{x \in \mathbb{R}^{N \times 2}} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{\alpha \in \mathbb{R}^{2 \times 1}} \simeq \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}}_{y \in \mathbb{R}^{N \times 1}}$$

```
import torch, numpy

data = torch.tensor(numpy.loadtxt('systolic-blood-pressure-vs-age.dat'))
nb_samples = data.size(0)

x, y = torch.empty(nb_samples, 2), torch.empty(nb_samples, 1)

x[:, 0] = data[:, 0]
x[:, 1] = 1

y[:, 0] = data[:, 1]

alpha, _ = torch.lstsq(y, x)

a, b = alpha[0, 0].item(), alpha[1, 0].item()
```


ax + b
data
240
220
200
180
160
140
120
100
Systolic blood presure (mmHg)
10
20
30
40
50
60
70
80
Age (years)