AMMI – Introduction to Deep Learning

3.3. Linear separability and feature design

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\[ \Phi : (x_u, x_v) \mapsto (x_u, x_v, x_u x_v). \]

So we can model the xor with

\[ f(x) = \sigma(w \Phi(x) + b). \]
Perceptron

\[ x \Phi \times w + b \sigma y \]
This is similar to the polynomial regression. If we have

\[ \Phi : x \mapsto (1, x, x^2, \ldots, x^D) \]

and

\[ \alpha = (\alpha_0, \ldots, \alpha_D) \]

then

\[ \sum_{d=0}^{D} \alpha_d x^d = \alpha \cdot \Phi(x). \]
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By increasing \( D \), we can approximate any continuous real function on a compact space (Stone-Weierstrass theorem).

It means that we can make the capacity as high as we want.
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Remember the bias-variance tradeoff.

$$\mathbb{E}((Y - y)^2) = (\mathbb{E}(Y) - y)^2 + \mathbb{V}(Y).$$

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In particular, good features should be invariant to perturbations of the signal known to keep the value to predict unchanged.
Training points
Votes (K=11)
Prediction (K=11)
Training points
Votes, radial feature ($K=11$)
Prediction, radial feature (K=11)
A classical example is the “Histogram of Oriented Gradient” descriptors (HOG), initially designed for person detection.

Roughly: divide the image in $8 \times 8$ blocks, compute in each the distribution of edge orientations over 9 bins.

Dalal and Triggs (2005) combined them with a SVM, and Dollár et al. (2009) extended them with other modalities into the “channel features”.
Many methods (perceptron, SVM, \textit{k}-means, PCA, etc.) only require to compute \( \kappa(x, x') = \Phi(x) \cdot \Phi(x') \) for any \((x, x')\).

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This is the **kernel trick**, which we will not talk about in this course.
Training a model composed of manually engineered features and a parametric model such as logistic regression is now referred to as “shallow learning”.

The signal goes through a single processing trained from data.
The end
References
