

EE-559 – Deep learning

3.3. Linear separability and feature design

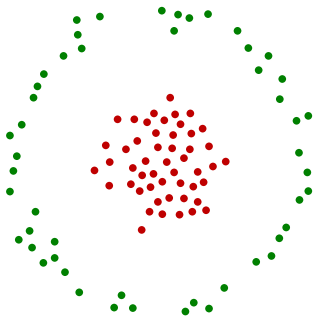
François Fleuret

<https://fleuret.org/ee559/>

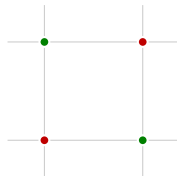
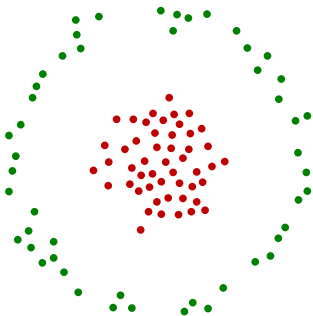
Wed May 1 15:08:28 UTC 2019

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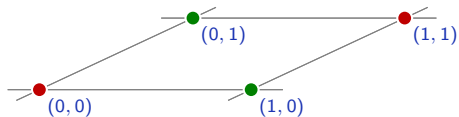


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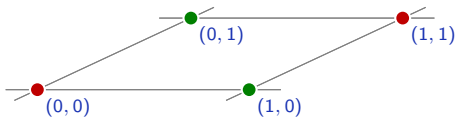
"xor"

The xor example can be solved by pre-processing the data to make the two populations linearly separable.



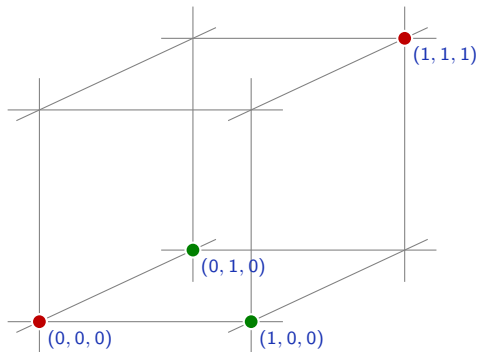
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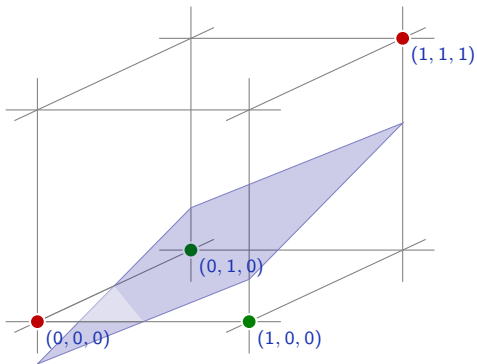
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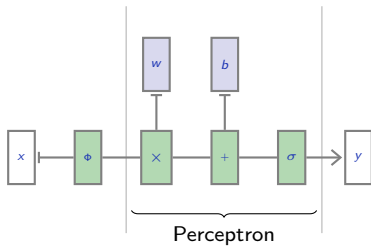
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This is similar to the polynomial regression. If we have

$$\Phi : x \mapsto (1, x, x^2, \dots, x^D)$$

and

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By increasing D , we can approximate any continuous real function on a compact space (Stone-Weierstrass theorem).

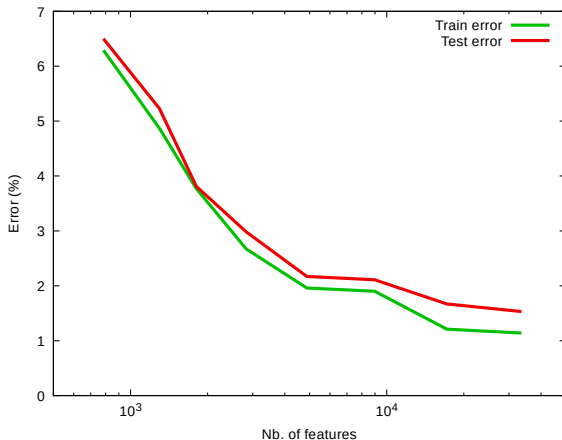
It means that we can make the capacity as high as we want.

We can apply the same to a more realistic binary classification problem: MNIST's "8" vs. the other classes with a perceptron.

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Remember the bias-variance tradeoff:

$$\mathbb{E}((Y - y)^2) = \underbrace{(\mathbb{E}(Y) - y)^2}_{\text{Bias}} + \underbrace{\mathbb{V}(Y)}_{\text{Variance}}.$$

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Beside increasing capacity to reduce the bias, “feature design” may also be a way of reducing capacity without hurting the bias, or with improving it.

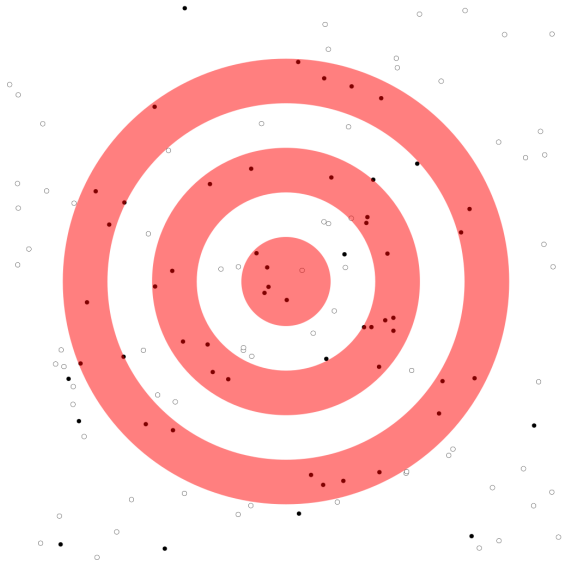
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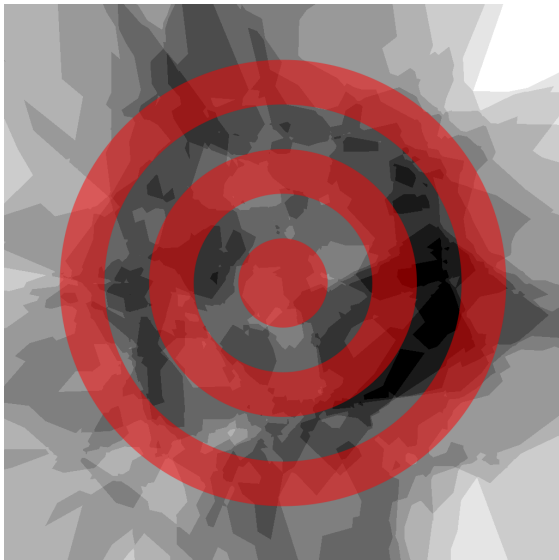
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In particular, good features should be invariant to perturbations of the signal known to keep the value to predict unchanged.



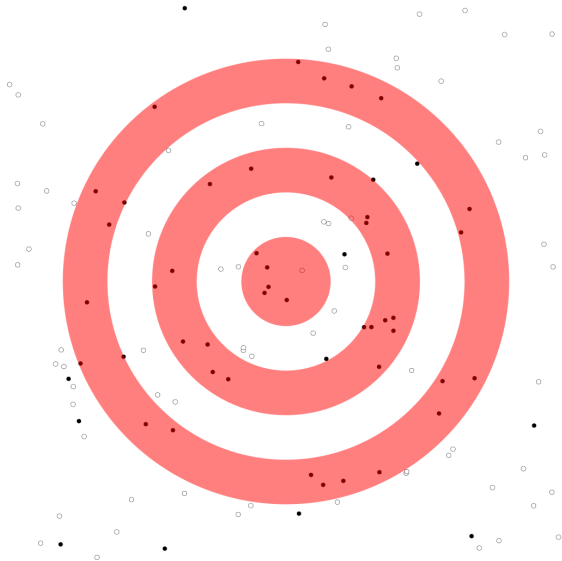
Training points



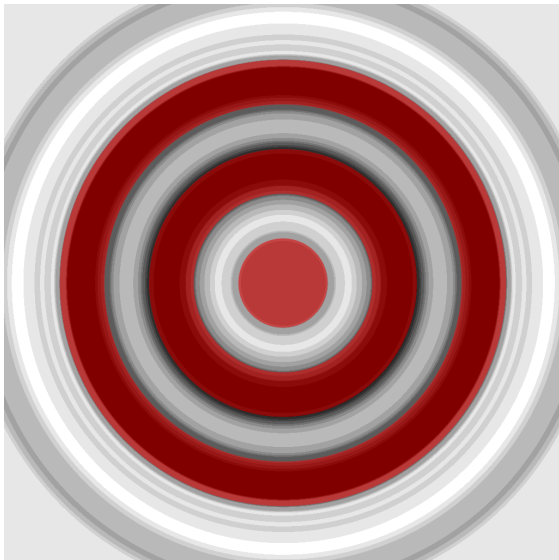
Votes (K=11)



Prediction ($K=11$)



Training points



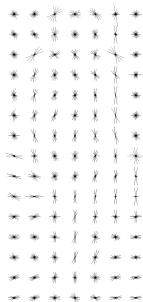
Votes, radial feature (K=11)



Prediction, radial feature (K=11)

A classical example is the “Histogram of Oriented Gradient” descriptors (HOG), initially designed for person detection.

Roughly: divide the image in 8×8 blocks, compute in each the distribution of edge orientations over 9 bins.



Dalal and Triggs (2005) combined them with a SVM, and Dollár et al. (2009) extended them with other modalities into the “channel features”.

Many methods (perceptron, SVM, k -means, PCA, etc.) only require to compute $\kappa(x, x') = \Phi(x) \cdot \Phi(x')$ for any (x, x') .

So one needs to specify κ alone, and may keep Φ undefined.

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This is the **kernel trick**, which we will not talk about in this course.

Training a model composed of manually engineered features and a parametric model such as logistic regression is now referred to as “**shallow learning**”.

The signal goes through a single processing trained from data.

The end

References

- N. Dalal and B. Triggs. Histograms of oriented gradients for human detection. In Conference on Computer Vision and Pattern Recognition (CVPR), pages 886–893, 2005.
- P. Dollár, Z. Tu, P. Perona, and S. Belongie. Integral channel features. In British Machine Vision Conference, pages 91.1–91.11, 2009.