

EE-559 – Deep learning

1.4. Tensor basics and linear regression

François Fleuret

<https://fleuret.org/ee559/>

Wed May 1 15:08:28 UTC 2019

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- A 3d tensor can be seen as a vector of identically sized matrix (e.g. a multi-channel image),
- A 4d tensor can be seen as a matrix of identically sized matrices, or a sequence of 3d tensors (e.g. a sequence of multi-channel images),
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Compounded data structures can represent more diverse data types.

PyTorch is a Python library built on top of Torch's THNN computational backend.

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A key specificity of PyTorch is the central role of autograd to compute derivatives of *anything!* We will come back to this.

```
>>> x = torch.empty(2, 5)
>>> x.size()
torch.Size([2, 5])
>>> x.fill_(1.125)
tensor([[ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250],
        [ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250]])
>>> x.mean()
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>>> x.std()
tensor(0.)
>>> x.sum()
tensor(11.2500)
>>> x.sum().item()
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In-place operations are suffixed with an underscore, and a 0d tensor can be converted back to a Python scalar with `item()`.

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Reading a coefficient also generates a 0d tensor.

```
>>> x = torch.tensor([[11., 12., 13.], [21., 22., 23.]])
>>> x[1, 2]
tensor(23.)
```

PyTorch provides operators for component-wise and vector/matrix operations.

```
>>> x = torch.tensor([ 10., 20., 30.])
>>> y = torch.tensor([ 11., 21., 31.])
>>> x + y
tensor([ 21., 41., 61.])
>>> x * y
tensor([ 110., 420., 930.])
>>> x**2
tensor([ 100., 400., 900.])
>>> m = torch.tensor([[ 0., 0., 3. ],
...                  [ 0., 2., 0. ],
...                  [ 1., 0., 0. ]])
>>> m.mv(x)
tensor([ 90., 40., 10.])
>>> m @ x
tensor([ 90., 40., 10.])
```

And as in `numpy`, the `:` symbol defines a range of values for an index and allows to slice tensors.

```
>>> import torch
>>> x = torch.empty(2, 4).random_(10)
>>> x
tensor([[8., 1., 1., 3.],
        [7., 0., 7., 5.]])
>>> x[0]
tensor([8., 1., 1., 3.])
>>> x[0, :]
tensor([8., 1., 1., 3.])
>>> x[:, 0]
tensor([8., 7.])
>>> x[:, 1:3] = -1
>>> x
tensor([[ 8., -1., -1.,  3.],
        [ 7., -1., -1.,  5.]])
```

PyTorch provides interfacing to standard linear operations, such as linear system solving or Eigen-decomposition.

```
>>> y = torch.empty(3).normal_()
>>> y
tensor([ 0.0477,  0.8834, -1.5996])
>>> m = torch.empty(3, 3).normal_()
>>> q, _ = torch.gels(y, m)
>>> torch.mm(m, q)
tensor([[ 0.0477],
        [ 0.8834],
        [-1.5996]])
```

Example: linear regression

Given a list of points

$$(x_n, y_n) \in \mathbb{R} \times \mathbb{R}, \quad n = 1, \dots, N,$$

can we find the “best line”

$$f(x; a, b) = ax + b$$

going “through the points”

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$$\operatorname{argmin}_{a,b} \frac{1}{N} \sum_{n=1}^N \left(\underbrace{ax_n + b}_{f(x_n; a, b)} - y_n \right)^2.$$

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Such a model would allow to predict the y associated to a new x , simply by calculating $f(x; a, b)$.

```
bash> cat systolic-blood-pressure-vs-age.dat
```

```
39 144
```

```
47 220
```

```
45 138
```

```
47 145
```

```
65 162
```

```
46 142
```

```
67 170
```

```
42 124
```

```
67 158
```

```
56 154
```

```
64 162
```

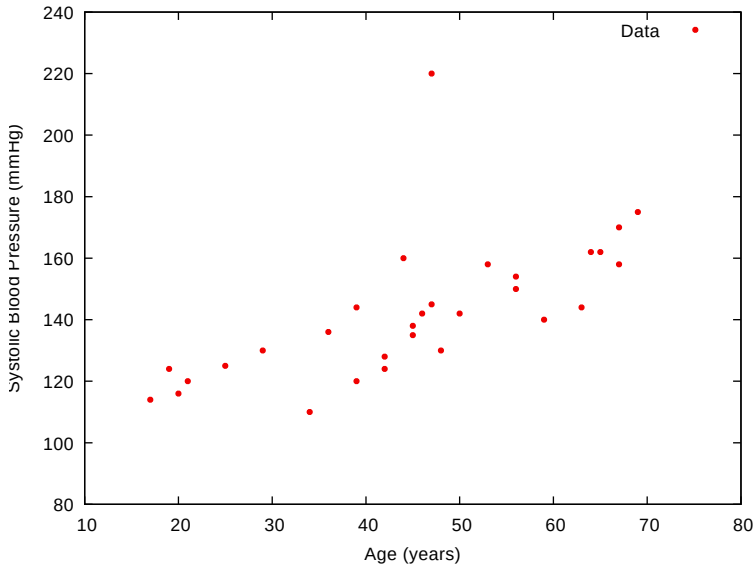
```
56 150
```

```
59 140
```

```
34 110
```

```
42 128
```

```
/.../
```



$$\underbrace{\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_N & y_N \end{pmatrix}}_{\text{data} \in \mathbb{R}^{N \times 2}}$$

$$\underbrace{\begin{pmatrix} x_1 & 1.0 \\ x_2 & 1.0 \\ \vdots & \vdots \\ x_N & 1.0 \end{pmatrix}}_{x \in \mathbb{R}^{N \times 2}} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{\alpha \in \mathbb{R}^{2 \times 1}} \approx \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}}_{y \in \mathbb{R}^{N \times 1}}$$

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```
import torch, numpy

data = torch.tensor(numpy.loadtxt('systolic-blood-pressure-vs-age.dat'))
nb_samples = data.size(0)

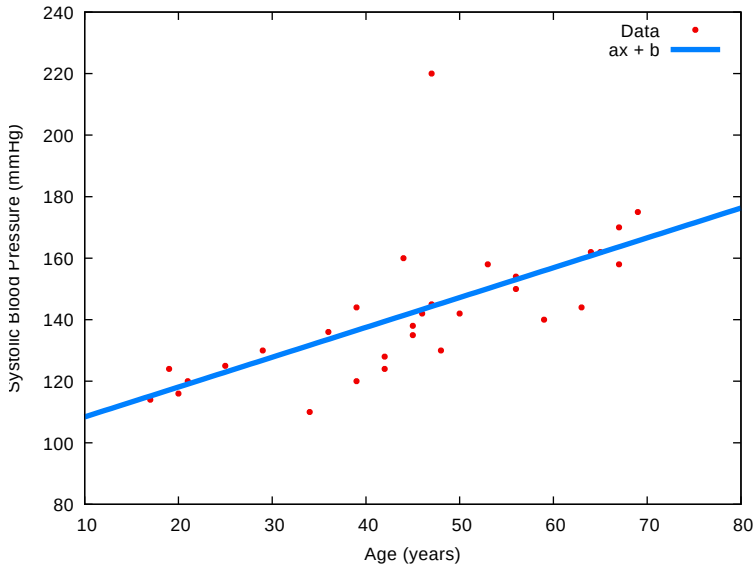
x, y = torch.empty(nb_samples, 2), torch.empty(nb_samples, 1)

x[:, 0] = data[:, 0]
x[:, 1] = 1

y[:, 0] = data[:, 1]

alpha, _ = torch.gels(y, x)

a, b = alpha[0, 0].item(), alpha[1, 0].item()
```



The end